UNIT 1

# Square Roots and Surface Area

## What You'll Learn

- Find square roots of fractions and decimals that are perfect squares.
- Approximate the square roots of fractions and decimals that are not perfect squares.
- Find the surface areas of composite objects.

## Why It's Important

Square roots are used by

- police officers, to estimate the speed of a vehicle when it crashed
- vets, to calculate drug dosages

Surface area is used by

- painters, to find the number of cans of paint needed to paint a room
- farmers, to find the amount of fertilizer needed for a field

## Key Words

square
square root
perfect square
non-perfect square
terminating decimal
repeating decimal
non-terminating, non-repeating decimal
surface area
composite object

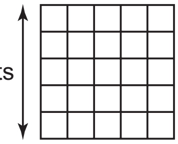
# 1.1 Skill Builder

## Side Lengths and Areas of Squares

The side length and area of a square are related.

- The area is the **square** of the side length.

5 units

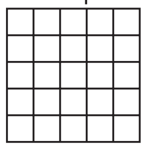


$$\begin{aligned} \text{Area} &= (\text{Length})^2 \\ &= 5^2 \\ &= 5 \times 5 \\ &= 25 \end{aligned}$$

The area is 25 square units.

- The side length is the **square root** of the area.

Area = 25 square units

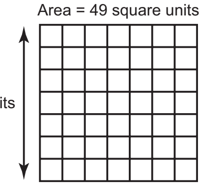
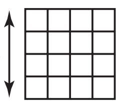
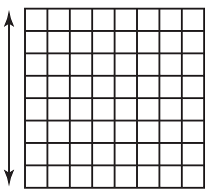
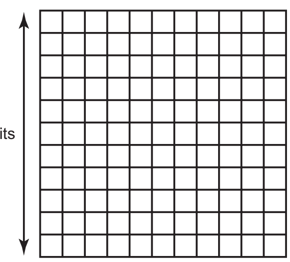


$$\begin{aligned} \text{Length} &= \sqrt{\text{Area}} \\ &= \sqrt{25} \\ &= \sqrt{5 \times 5} \\ &= 5 \end{aligned}$$

The side length is 5 units.

## *Check*

**1.** Which square and square root are modelled by each diagram?

| Diagram | Square Modelled | Square Root Modelled |
|---|---|---|
| **a)** Area = 49 square units  7 units | $(\text{Length})^2 = \text{Area}$<br>$7^2 =$ _____<br>The area is 49 square units. | $\sqrt{\text{Area}} = \text{Length}$<br>$\sqrt{49} =$ _____<br>The side length is 7 units. |
| **b)**  4 units | _____ = _____<br>The area is _____ square units. | $\sqrt{\phantom{xx}}$ _____ = _____<br>The side length is _____ units. |
| **c)**  8 units | _____ = _____<br>The area is _____ square units. | $\sqrt{\phantom{xx}}$ _____ = _____<br>The side length is _____ units. |
| **d)**  11 units | _____ = _____<br>The area is _____ square units. | $\sqrt{\phantom{xx}}$ _____ = _____<br>The side length is _____ units. |

## Whole Number Squares and Square Roots

- The square of a number is the number multiplied by itself.
- A square root of a number is one of 2 equal factors of the number.
- Squaring and taking a square root are inverse operations.

$5^2 = 5 \times 5$
$= 25$
$\sqrt{25} = \sqrt{5 \times 5}$
$= 5$
$5^2 = 25$ and $\sqrt{25} = 5$

### *Check*

**1.** Complete each sentence.

**a)** $4^2 = 16$, so $\sqrt{16} =$ ____

**b)** $12^2 =$ ____, so $\sqrt{\_\_\_\_} =$ ____

**c)** $\sqrt{25} =$ ____, since ____ $= 25$

**d)** $\sqrt{100} =$ ____, since ____ $=$ ____

## Perfect Squares

A number is a **perfect square** if it is the product of 2 equal factors.
25 is a perfect square because $25 = 5 \times 5$.
24 is a **non-perfect square.** It is not the product of 2 equal factors.

### *Check*

**1.** Complete each sentence.

| First 12 Whole-Number Perfect Squares | | | |
|---|---|---|---|
| **Perfect Square** | **Square Root** | **Perfect Square** | **Square Root** |
| $1^2 = 1 \times 1 = 1$ | $\sqrt{1} = 1$ | $7^2 =$ ___ × ___ = ___ | $\sqrt{\_\_\_} =$ ___ |
| $2^2 = 2 \times 2 = 4$ | $\sqrt{4} = 2$ | $8^2 =$ ___ × ___ = ___ | $\sqrt{\_\_\_} =$ ___ |
| $3^2 =$ ___ × ___ = ___ | $\sqrt{\_\_\_} =$ ___ | $9^2 =$ ___ × ___ = ___ | $\sqrt{\_\_\_} =$ ___ |
| $4^2 =$ ___ × ___ = ___ | $\sqrt{\_\_\_} =$ ___ | $10^2 =$ ___ × ___ = ___ | $\sqrt{\_\_\_} =$ ___ |
| $5^2 =$ ___ × ___ = ___ | $\sqrt{\_\_\_} =$ ___ | $11^2 =$ ___ × ___ = ___ | $\sqrt{\_\_\_} =$ ___ |
| $6^2 =$ ___ × ___ = ___ | $\sqrt{\_\_\_} =$ ___ | $12^2 =$ ___ × ___ = ___ | $\sqrt{\_\_\_} =$ ___ |

# 1.1 Square Roots of Perfect Squares

**FOCUS** **Find the square roots of decimals and fractions that are perfect squares.**

The square of a fraction or decimal is the number multiplied by itself.

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2}{3 \times 3} = \frac{4}{9}$$

$$(1.5)^2 = 1.5 \times 1.5 = 2.25$$

$\frac{4}{9}$ and 2.25 are perfect squares because they are the product of 2 equal factors.

$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, so

$\frac{2}{3}$ is a square root of $\frac{4}{9}$.

We write: $\sqrt{\frac{4}{9}} = \frac{2}{3}$

$2.25 = 1.5 \times 1.5$, so

1.5 is a square root of 2.25.

We write: $\sqrt{2.25} = 1.5$

*Each equal factor is a square root of the perfect square.*

## Example 1 Finding a Perfect Square Given Its Square Root

Calculate the number whose square root is:

**a)** $\frac{5}{8}$

**b)** 1.2

### Solution

A square root of a number is one of two equal factors of the number.

**a)** $\frac{5}{8}$

$$\frac{5}{8} \times \frac{5}{8} = \frac{5 \times 5}{8 \times 8} = \frac{25}{64}$$

So, $\frac{5}{8}$ is a square root of $\frac{25}{64}$.

**b)** 1.2

Use a calculator.

$1.2 \times 1.2 = 1.44$

So, 1.2 is a square root of 1.44.

## *Check*

**1.** Calculate the perfect square with the given square root.

**a)** $\frac{3}{8}$

$\frac{3}{8} \times \frac{3}{8} = \frac{\_\_\_\_ \times \_\_\_\_}{\_\_\_\_ \times \_\_\_\_}$

$= \frac{\_\_\_\_}{\_\_\_\_}$

$\frac{3}{8}$ is a square root of ______.

**b)** $\frac{3}{2}$

$\frac{\_\_\_}{\_\_\_} \times \frac{\_\_\_}{\_\_\_} = \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$

$= \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$

$\frac{3}{2}$ is a square root of ______.

**c)** 0.5

$0.5 \times 0.5 =$ ______

0.5 is a square root of ______.

**d)** 2.5

$2.5 \times 2.5 =$ ______

2.5 is a square root of ______.

## *Example 2* Identifying Fractions that Are Perfect Squares

Is each fraction a perfect square? If so, find its square root.

**a)** $\frac{16}{25}$

**b)** $\frac{9}{20}$

### *Solution*

Check if the numerator and denominator are perfect squares.

**a)** $\frac{16}{25}$

$16 = 4 \times 4$, so 16 is a perfect square.

$25 = 5 \times 5$, so 25 is a perfect square.

So, $\frac{16}{25}$ is a perfect square.

**b)** $\frac{9}{20}$

$9 = 3 \times 3$, so 9 is a perfect square.

20 is not a perfect square.

So, $\frac{9}{20}$ is not a perfect square.

## *Check*

**1.** Determine whether the fraction is or is not a perfect square. How do you know?

**a)** $\frac{9}{49}$

9 ________ a perfect square because ________________________________________.

49 ________ a perfect square because ________________________________________.

So, $\frac{9}{49}$ ________ a perfect square.

**b)** $\frac{25}{13}$

25 ________ a perfect square because ________________________________________.

13 ________ a perfect square because ________________________________________.

So, $\frac{25}{13}$ ________ a perfect square.

**c)** $\frac{64}{81}$ 64 ________ a perfect square because ______________________________.

81 ________ a perfect square because ______________________________.

So, $\frac{64}{81}$ ________ a perfect square.

**2.** Find the value of each square root.

**a)** $\sqrt{\frac{9}{4}} = \sqrt{\frac{\_\_\_\_ \times \_\_\_\_}{\_\_\_\_ \times \_\_\_\_}} = \frac{\_\_\_\_}{\_\_\_\_}$

**b)** $\sqrt{\frac{16}{81}} = \sqrt{\frac{\_\_\_\_ \times \_\_\_\_}{\_\_\_\_ \times \_\_\_\_}} = \frac{\_\_\_\_}{\_\_\_\_}$

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A **terminating decimal** ends after a certain number of decimal places.
A **repeating decimal** has a repeating pattern of digits in the decimal expansion.
The bar shows the digits that repeat.

| Terminating | Repeating | Non-terminating and non-repeating |
|---|---|---|
| 0.5 0.28 | $0.333\ 333 \ldots = 0.\overline{3}$<br>$0.191\ 919 \ldots = 0.\overline{19}$ | 1.414 213 56 … 7.071 067 812 … |

You can use a calculator to find out if a decimal is a perfect square.
The square root of a perfect square decimal is either a terminating decimal or a repeating decimal.

## *Example 3* Identifying Decimals that Are Perfect Squares

Is each decimal a perfect square? How do you know?

**a)** 1.69 **b)** 3.5

### *Solution*

Use a calculator to find the square root of each number.

**a)** $\sqrt{1.69} = 1.3$
The square root is the terminating decimal 1.3.
So, 1.69 is a perfect square.

**b)** $\sqrt{3.5} \doteq 1.870\ 828\ 693$
The square root appears to be a decimal that neither repeats nor terminates.
So, 3.5 is not a perfect square.

*The symbol $\doteq$ means "approximately equal to".*

## *Check*

**1.** Complete the table to find whether each decimal is a perfect square.
The first one is done for you.

| | Decimal | Value of square root | Type of decimal | Is decimal a perfect square? |
|---|---|---|---|---|
| **a)** | 70.5 | 8.396 427 811 … | Non-repeating<br>Non-terminating | No |
| **b)** | 5.76 | ________ | ________<br>________ | ________ |
| **c)** | 0.25 | ________ | ________<br>________ | ________ |
| **d)** | 2.5 | ________ | ________<br>________ | ________ |

## Practice

**1.** Calculate the number whose square root is:

**a)** $\frac{1}{4}$

$\frac{1}{4} \times \frac{1}{4} = \frac{\_\_ \times \_\_}{\_\_ \times \_\_}$

$= \frac{\_\_}{\_\_}$

$\frac{1}{4}$ is a square root of ____.

**b)** $\frac{2}{7}$

$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_ \times \_\_}{\_\_ \times \_\_}$

$= \frac{\_\_}{\_\_}$

$\frac{2}{7}$ is a square root of ____.

**c)** 0.6

_____ × _____ = _____

0.6 is a square root of _____.

**d)** 1.1

_____ × _____ = _____

1.1 is a square root of _____.

**2.** Identify the fractions that are perfect squares. The first one has been done for you.

| | Fraction | Is numerator a perfect square? | Is denominator a perfect square? | Is fraction a perfect square? |
|---|---|---|---|---|
| **a)** | $\frac{81}{125}$ | Yes; $9 \times 9 = 81$ | No | No |
| **b)** | $\frac{25}{49}$ | ________ | ________ | ________ |
| **c)** | $\frac{36}{121}$ | ________ | ________ | ________ |
| **d)** | $\frac{17}{25}$ | ________ | ________ | ________ |
| **e)** | $\frac{9}{100}$ | ________ | ________ | ________ |

**3.** Find each square root.

**a)** $\sqrt{\frac{49}{100}} = \sqrt{\frac{\underline{\quad} \times \underline{\quad}}{\underline{\quad} \times \underline{\quad}}}$

$= \frac{\underline{\quad}}{\underline{\quad}}$

**b)** $\sqrt{\frac{25}{144}} = \sqrt{\frac{\underline{\quad} \times \underline{\quad}}{\underline{\quad} \times \underline{\quad}}}$

$= \frac{\underline{\quad}}{\underline{\quad}}$

**c)** $\sqrt{\frac{1}{16}} = \sqrt{\frac{\underline{\quad} \times \underline{\quad}}{\underline{\quad} \times \underline{\quad}}}$

$= \frac{\underline{\quad}}{\underline{\quad}}$

**d)** $\sqrt{\frac{9}{400}} = \sqrt{\frac{\underline{\quad} \times \underline{\quad}}{\underline{\quad} \times \underline{\quad}}}$

$= \frac{\underline{\quad}}{\underline{\quad}}$

**4.** Use a calculator. Find each square root.

**a)** $\sqrt{8.41} =$ _____ **b)** $\sqrt{0.0676} =$ _____ **c)** $\sqrt{51.125} =$ _____ **d)** $\sqrt{6.25} =$ _____

**5.** Which decimals are perfect squares?

**a)** 1.44

$\sqrt{1.44} =$ ______________

The square root is a decimal that ______________________________________.

So, 1.44 _______ a perfect square.

**b)** 30.25

$\sqrt{30.25} =$ ______________

The square root is a decimal that ______________________________________.

So, 30.25 _______ a perfect square.

**c)** 8.5

$\sqrt{8.5} \doteq$ ______________

The square root is a decimal that ______________________________________.

So, 8.5 _______ a perfect square.

**d)** 0.0256

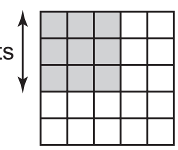
$\sqrt{0.0256} =$ ______________

The square root is a decimal that ______________________________________.

So, 0.0256 _______ a perfect square.

**6.** Find the area of each square.

**a)** $\frac{\underline{\quad}}{\underline{\quad}}$ units

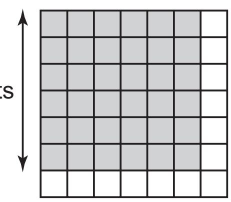


Area = ________

= ________

= ________

**b)** $\frac{\underline{\quad}}{\underline{\quad}}$ units



Area = ________

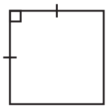
= ________

= ________

Area = $(\text{Length})^2$

The area is ____________________ ____________________________________

**c)** 5.4 units



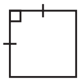
Area = ______

= ______ × ______

= ________

______________________________

**d)** 2.1 units



Area = ______

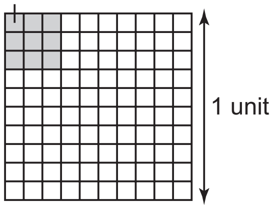
= ______ × ______

= ________

______________________________

**7.** Find the side length of each square.

**a)** Area = $\frac{9}{100}$ square units




Side Length = $\sqrt{\qquad}$

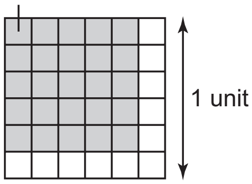
= $\sqrt{\qquad}$

= ________

Length = $\sqrt{\text{Area}}$

The side length is ___ units.

**b)** Area = $\frac{25}{36}$ square units



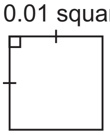

Length = $\sqrt{\qquad}$

= $\sqrt{\qquad}$

= ________

______________________________

**c)** Area = 0.01 square units

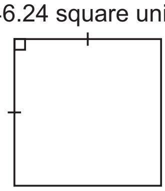


Length = $\sqrt{\qquad}$

= ________

______________________________

**d)** Area =
46.24 square units



Length = $\sqrt{\qquad}$

= ________

______________________________

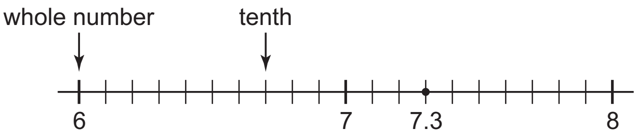
# 1.2 Skill Builder

## Degree of Accuracy

We are often asked to write an answer to a given decimal place.
To do this, we can use a number line.

To write 7.3 to the nearest whole number:
Place 7.3 on a number line in tenths.

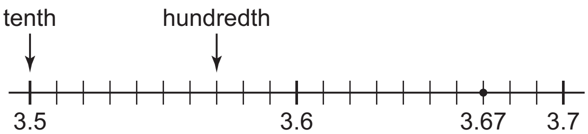
*3 is the last digit. It is in the tenths position. So, use a number line in tenths.*




7.3 is closer to 7 than to 8.
So, 7.3 to the nearest whole number is: 7

To write 3.67 to the nearest tenth:
Place 3.67 on a number line in hundredths.

*7 is the last digit. It is in the hundredths position. So, use a number line in hundredths.*

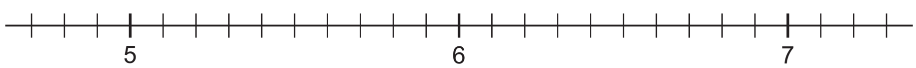



3.67 is closer to 3.7 than to 3.6.
So, 3.67 to the nearest tenth is: 3.7

### *Check*

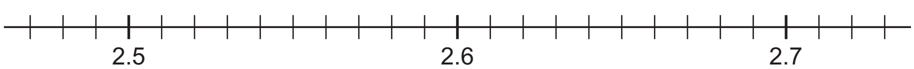
**1.** Write each number to the nearest whole number.
Mark it on the number line.

**a)** 5.3 _____ **b)** 6.8 _____ **c)** 7.1 _____ **d)** 6.4 _____



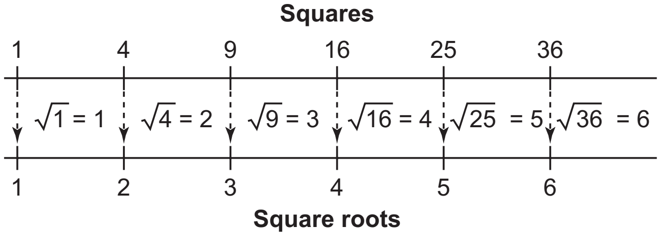

**2.** Write each number to the nearest tenth.
Mark it on the number line.

**a)** 2.53 _____ **b)** 2.64 _____ **c)** 2.58 _____ **d)** 2.66 _____

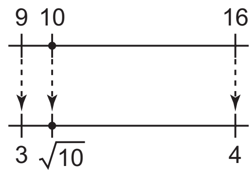


## Squares and Square Roots on Number Lines

Most numbers are not perfect squares.
You can use number lines to estimate the square roots of these numbers.




10 is between the perfect squares 9 and 16.
So, $\sqrt{10}$ is between $\sqrt{9}$ and $\sqrt{16}$.
$\sqrt{9} = 3$ and $\sqrt{16} = 4$
So, $\sqrt{10}$ is between 3 and 4.




*10 is closer to 9 than 16, so $\sqrt{10}$ is closer to 3 than 4.*

Check with a calculator.
$\sqrt{10} \doteq 3.2$, which is between 3 and 4.

## *Check*

**1.** Between which 2 consecutive whole numbers is each square root? Explain.

*Refer to the squares and square roots number lines.*

**a)** $\sqrt{22}$

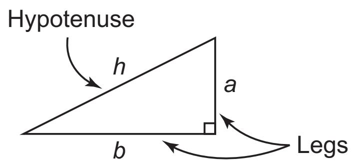
22 is between the perfect squares 16 and 25.
So, $\sqrt{22}$ is between $\sqrt{\_\_\_\_\_}$ and $\sqrt{\_\_\_\_\_}$.
$\sqrt{\_\_\_\_\_}$ = _____ and $\sqrt{\_\_\_\_\_}$ = _____
So, $\sqrt{22}$ is between _____ and _____.

**b)** $\sqrt{6}$

6 is between the perfect squares _____ and _____.
So, $\sqrt{6}$ is between $\sqrt{\_\_\_\_\_}$ and $\sqrt{\_\_\_\_\_}$.
$\sqrt{\_\_\_\_\_}$ = _____ and $\sqrt{\_\_\_\_\_}$ = _____
So, $\sqrt{6}$ is between _____ and _____.

## The Pythagorean Theorem

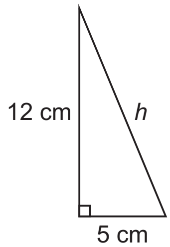
You can use the Pythagorean Theorem to find unknown lengths in right triangles.




**Pythagorean Theorem**

$h^2 = a^2 + b^2$

To find the length of the hypotenuse, *h*, in this triangle:



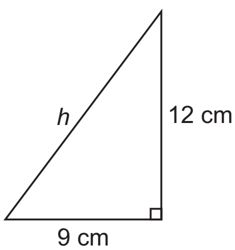

$$h^2 = 5^2 + 12^2$$
$$h^2 = 25 + 144$$
$$h^2 = 169$$
$$h = \sqrt{169}$$
$$h = 13$$

The length of the hypotenuse is 13 cm.

## *Check*

**1.** Use the Pythagorean Theorem to find the length of each hypotenuse, *h*.

**a)**




$h^2 =$ _____ + _____

$h^2 =$ _____ + _____

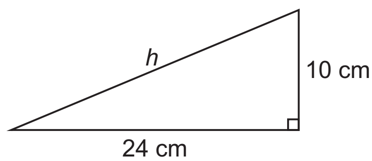
$h^2 =$ _____

$h = \sqrt{\phantom{000}}$ _____

$h =$ _____

The length of the hypotenuse is _____ cm.

**b)**




$h^2 =$ _____ + _____

$h^2 =$ _____ + _____

$h^2 =$ _____

$h = \sqrt{\phantom{000}}$ _____

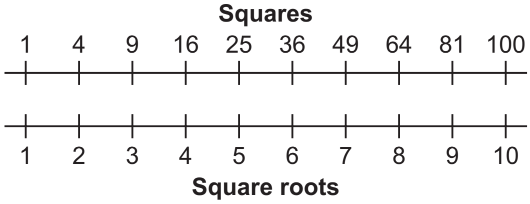
$h =$ _____

The length of the hypotenuse is _____ cm.

# 1.2 Square Roots of Non-Perfect Squares

**FOCUS** **Approximate the square roots of decimals and fractions that are not perfect squares.**

The top number line shows all the perfect squares from 1 to 100.




The bottom number line shows the square root of each number in the top line. You can use these lines to estimate the square roots of fractions and decimals that are not perfect squares.

## *Example 1* Estimating a Square Root of a Decimal

Estimate: $\sqrt{68.5}$

### *Solution*

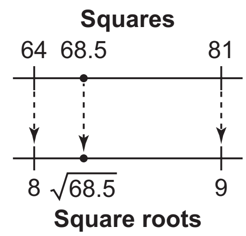
68.5 is between the perfect squares 64 and 81.

So, $\sqrt{68.5}$ is between $\sqrt{64}$ and $\sqrt{81}$.

That is, $\sqrt{68.5}$ is between 8 and 9.

Since 68.5 is closer to 64 than 81, $\sqrt{68.5}$ is closer to 8 than 9.

So, $\sqrt{68.5}$ is between 8 and 9, and closer to 8.




### *Check*

**1.** Estimate each square root.
Explain your estimate.

**a)** $\sqrt{13.5}$

13.5 is between the perfect squares ____ and ____.

So, $\sqrt{13.5}$ is between $\sqrt{\phantom{00}}$ and $\sqrt{\phantom{00}}$.

That is, $\sqrt{13.5}$ is between ____ and ____.

Since 13.5 is closer to ____ than ____, $\sqrt{13.5}$ is closer to ____ than ____.

So, $\sqrt{13.5}$ is between ____ and ____, and closer to ____.

**b)** $\sqrt{51.5}$

51.5 is between the perfect squares ______ and ______.

So, $\sqrt{51.5}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.

That is, $\sqrt{51.5}$ is between ____ and ____.

Since 51.5 is closer to ____ than ____, $\sqrt{51.5}$ is closer to ____ than ____.

So, $\sqrt{51.5}$ is between ____ and ____, and closer to ____.

## *Example 2* Estimating a Square Root of a Fraction

Estimate: $\sqrt{\frac{3}{10}}$

### *Solution*

Find the closest perfect square to the numerator and denominator.

In the fraction $\frac{3}{10}$:

3 is close to the perfect square 4.

10 is close to the perfect square 9.

So, $\sqrt{\frac{3}{10}} \doteq \sqrt{\frac{4}{9}}$ and $\sqrt{\frac{4}{9}} = \frac{2}{3}$

So, $\sqrt{\frac{3}{10}} \doteq \frac{2}{3}$

## *Check*

**1.** Estimate each square root.

**a)** $\sqrt{\frac{23}{80}}$

23 is close to the perfect square ____.

80 is close to the perfect square ____.

So, $\sqrt{\frac{23}{80}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\sqrt{\frac{\_\_\_\_}{\_\_\_\_}} = \frac{\_\_\_}{\_\_\_}$

So, $\sqrt{\frac{23}{80}} \doteq$ ___

**b)** $\sqrt{\frac{8}{17}}$

8 is close to the perfect square ____.

17 is close to the perfect square ____.

So, $\sqrt{\frac{8}{17}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\sqrt{\frac{\_\_\_\_}{\_\_\_\_}} = \frac{\_\_\_}{\_\_\_}$

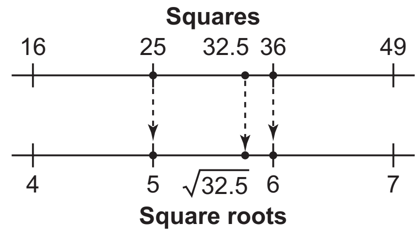
So, $\sqrt{\frac{8}{17}} \doteq$ ______

## *Example 3* Finding a Number with a Square Root between Two Given Numbers

Identify a decimal that has a square root between 5 and 6.

### *Solution*

$5^2 = 25$, so 5 is a square root of 25.
$6^2 = 36$, so 6 is a square root of 36.
So, any decimal between 25 and 36 has a square root between 5 and 6.
Choose 32.5.




Check the answer by using a calculator.
$\sqrt{32.5} \doteq 5.7$, which is between 5 and 6.
So, the decimal 32.5 is one correct answer.
There are many more correct answers.

### *Check*

**1. a)** Identify a decimal that has a square root between 7 and 8.
Check the answer.
$7^2 =$ _____ and $8^2 =$ _____
So, any decimal between _____ and _____ has a square root between 7 and 8.
Choose _____.
Check the answer on a calculator.
$\sqrt{\_\_\_\_\_} \doteq$ _____
The decimal _____ is one correct answer.

**b)** Identify a decimal that has a square root between 11 and 12.
_____ = _____ and _____ = _____
So, any decimal between _____ and _____ has a square root between 11 and 12.
Choose ______.
$\sqrt{\_\_\_\_\_\_} \doteq$ _____
So, ______ is one correct answer.

## Practice

**1.** For each number, name the 2 closest perfect squares and their square roots.

| | Number | Two closest perfect squares | Their square roots |
|---|---|---|---|
| **a)** | 44.4 | ____ and ____ | ____ and ____ |
| **b)** | 10.8 | ____ and ____ | ____ and ____ |
| **c)** | 125.9 | ____ and ____ | ____ and ____ |
| **d)** | 87.5 | ____ and ____ | ____ and ____ |

**2.** For each fraction, name the closest perfect square and its square root for the numerator and for the denominator.

| | Fraction | Closest perfect squares | Their square roots |
|---|---|---|---|
| **a)** | $\frac{5}{11}$ | Numerator: ____; denominator: ____ | ____ and ____ |
| **b)** | $\frac{17}{45}$ | Numerator: ____; denominator: ____ | ____ and ____ |
| **c)** | $\frac{3}{24}$ | Numerator: ____; denominator: ____ | ____ and ____ |
| **d)** | $\frac{11}{62}$ | Numerator: ____; denominator: ____ | ____ and ____ |

**3.** Estimate each square root.
Explain.

**a)** $\sqrt{1.6}$

1.6 is between ____ and ____.
So, $\sqrt{1.6}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.
That is, $\sqrt{1.6}$ is between ____ and ____.
Since 1.6 is closer to ____ than ____, $\sqrt{1.6}$ is closer to ____ than ____.
So, $\sqrt{1.6}$ is between ____ and ____, and closer to ____.

**b)** $\sqrt{44.5}$

44.5 is between ____ and ____.
So, $\sqrt{44.5}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.
That is, $\sqrt{44.5}$ is between ____ and ____.
Since 44.5 is closer to ____ than ____, $\sqrt{44.5}$ is closer to ____ than ____.
So, $\sqrt{44.5}$ is between ____ and ____, and closer to ____.

**c)** $\sqrt{75.8}$

75.8 is between ____ and ____.

So, $\sqrt{75.8}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.

That is, $\sqrt{75.8}$ is between ____ and ____.

Since 75.8 is closer to ____ than ____, $\sqrt{75.8}$ is closer to ____ than ____.

So, $\sqrt{75.8}$ is between ____ and ____, and closer to ____.

**4.** Estimate each square root. Explain.

**a)** $\sqrt{\frac{7}{15}}$

7 is close to ____; 15 is close to ____.

So, $\sqrt{\frac{7}{15}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\doteq$ ____

**b)** $\sqrt{\frac{2}{7}}$

2 is close to ____; 7 is close to ____.

So, $\sqrt{\frac{2}{7}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\doteq$ ____

**c)** $\sqrt{\frac{35}{37}}$

35 is close to ____; 37 is close to ____.

So, $\sqrt{\frac{35}{37}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\doteq$ ________

**d)** $\sqrt{\frac{99}{122}}$

99 is close to ____; 122 is close to ____.

So, $\sqrt{\frac{99}{122}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\doteq$ ________

**5.** Identify a decimal that has a square root between the two given numbers. Check the answer.

**a)** 1 and 2

$1^2 =$ _____ and $2^2 =$ _____

So, any number between _____ and _____ has a square root between 1 and 2.

Choose _____.

Check: $\sqrt{\_\_\_\_\_} \doteq$ _____

The decimal _____ is one possible answer.

**b)** 8 and 9

$8^2 =$ _____ and $9^2 =$ _____

So, any number between _____ and _____ has a square root between 8 and 9.

Choose _____.

Check: $\sqrt{\_\_\_\_\_} \doteq$ _____

The decimal _____ is one possible answer.

**c)** 2.5 and 3.5

________ = ________ and ________ = ________

So, any number between ________ and ________ has a square root between 2.5 and 3.5.

Choose ________.

Check: $\sqrt{\_\_\_\_\_\_\_\_} \doteq$ ________

The decimal ________ is one correct answer.

**d)** 20 and 21

________ = ________ and ________ = ________

So, any number between ________ and ________ has a square root between 20 and 21.

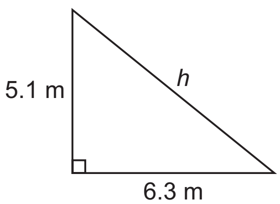
Choose ________.

Check: $\sqrt{\_\_\_\_\_\_\_\_} \doteq$ ________

The decimal ________ is one correct answer.

**6.** Determine the length of the hypotenuse in each right triangle.
Write each answer to the nearest tenth.

**a)**




$h^2 = 5.1^2 + 6.3^2$

$h^2 =$ ______ + ______

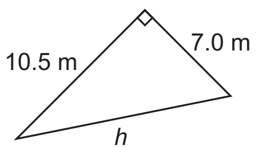
$h^2 =$ ______

$h = \sqrt{\_\_\_\_\_\_}$

$h \doteq$ ______

So, $h$ is about ______ m.

**b)**




$h^2 =$ ________ + ________

$h^2 =$ ________ + ________

$h^2 =$ ________

$h = \sqrt{\_\_\_\_\_\_\_\_}$

$h \doteq$ ________

So, $h$ is about ________ m.

CHECKPOINT

## *Can you ...*

- Identify decimals and fractions that are perfect squares?
- Find the square roots of decimals and fractions that are perfect squares?
- Approximate the square roots of decimals and fractions that are not perfect squares?

**1.1** **1.** Calculate the number whose square root is:

**a)** $\frac{2}{7}$

$\frac{2}{7} \times \frac{2}{7} = \frac{\_\_\_\_}{}$

$\frac{2}{7}$ is a square root of ____.

**b)** $\frac{8}{11}$

$\frac{}{\_\_\_\_} \times \frac{}{\_\_\_\_} = \frac{}{\_\_\_\_}$

$\frac{8}{11}$ is a square root of ____.

**c)** 0.1

____ × ____ = ____

0.1 is a square root of ____.

**d)** 1.4

1.4 × 1.4 = ____

1.4 is a square root of ____.

**2.** Identify the fractions that are perfect squares.
The first one has been done for you.

| | Fraction | Is numerator a perfect square? | Is denominator a perfect square? | Is fraction a perfect square? |
|---|---|---|---|---|
| **a)** | $\frac{64}{75}$ | Yes; 8 × 8 = 64 | No | No |
| **b)** | $\frac{9}{25}$ | ____ | ____ | ____ |
| **c)** | $\frac{25}{55}$ | ____ | ____ | ____ |

**3.** Find each square root.

**a)** $\sqrt{\frac{9}{49}} = \sqrt{\frac{\_\_\_\_ \times \_\_\_\_}{\_\_\_\_ \times \_\_\_\_}}$
$= \frac{}{\_\_\_\_}$

**b)** $\sqrt{\frac{16}{25}} = \sqrt{\frac{\_\_\_\_ \times \_\_\_\_}{\_\_\_\_ \times \_\_\_\_}}$
$= \frac{}{\_\_\_\_}$

**c)** $\sqrt{\frac{36}{121}} = \sqrt{\frac{\_\_\_\_ \times \_\_\_\_}{\_\_\_\_ \times \_\_\_\_}}$
$= \frac{}{\_\_\_\_}$

**4. a)** Put a check mark beside each decimal that is a perfect square.

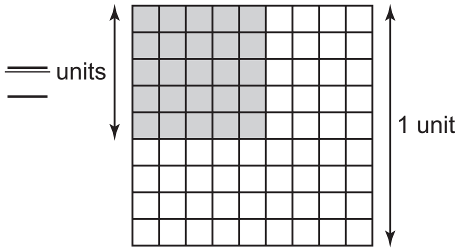
**i)** 4.84 ____ **ii)** 3.63 ____ **iii)** 98.01 ____ **iv)** 67.24 ____

**b)** Explain how you identified the perfect squares in part a.

____________________________________________

____________________________________________

**5. a)** Find the area of the shaded square.




Area = (Length)$^2$

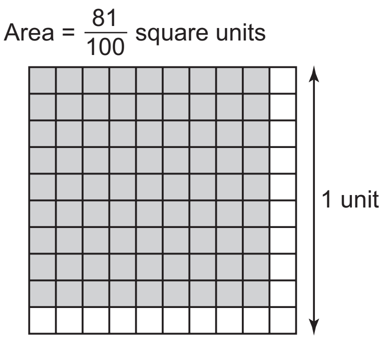
$= \left(\frac{\_\_}{\_\_}\right)^2$

$= \frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_}$

$= \frac{\_\_\_}{\_\_\_}$

The area is _____ square units.

**b)** Find the side length of the shaded square.




Length = $\sqrt{\text{Area}}$

$= \sqrt{\frac{\_\_\_}{\_\_\_}}$

$= \sqrt{\frac{\_\_ \times \_\_}{\_\_ \times \_\_}}$

$= \frac{\_\_\_}{\_\_\_}$

The side length is _____ units.

**1.2** **6.** Estimate each square root.
Explain.

**a)** $\sqrt{7.5}$

7.5 is between _____ and _____.
So, $\sqrt{7.5}$ is between $\sqrt{\_\_\_\_\_}$ and $\sqrt{\_\_\_\_\_}$.
That is, $\sqrt{7.5}$ is between _____ and _____.
Since 7.5 is closer to _____ than _____, $\sqrt{7.5}$ is closer to _____ than _____.
So, $\sqrt{7.5}$ is between _____ and _____, and closer to _____.

**b)** $\sqrt{66.6}$

66.6 is between _____ and _____.
So, $\sqrt{66.6}$ is between $\sqrt{\_\_\_\_\_}$ and $\sqrt{\_\_\_\_\_}$.
That is, $\sqrt{66.6}$ is between _____ and _____.
Since 66.6 is closer to _____ than _____, $\sqrt{66.6}$ is closer to _____ than _____.
So, $\sqrt{66.6}$ is between _____ and _____, and closer to _____.

**7.** Estimate each square root.

**a)** $\sqrt{\frac{15}{79}}$

15 is close to ____; 79 is close to ____.

So, $\sqrt{\frac{15}{79}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\doteq \frac{\_\_\_\_}{\_\_\_\_}$

**b)** $\sqrt{\frac{23}{50}}$

23 is close to ____; 50 is close to ____.

So, $\sqrt{\frac{23}{50}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\doteq \frac{\_\_\_\_}{\_\_\_\_}$

**8.** Identify a decimal whose square root is between the given numbers. Check your answer.

**a)** 2 and 3

$2^2 =$ _____ and $3^2 =$ _____

So, any number between _____ and _____ has a square root between 2 and 3.

Choose _____.

Check: $\sqrt{\_\_\_\_} \doteq$ _____

The decimal _____ is one correct answer.

**b)** 6 and 7

$6^2 =$ _____ and $7^2 =$ _____

So, any number between _____ and _____ has a square root between 6 and 7.

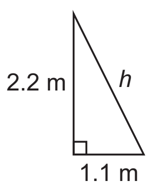
Choose _____.

$\sqrt{\_\_\_\_} \doteq$ _____

The decimal _____ is one correct answer.

**9.** Find the length of each hypotenuse.

**a)**




$h^2 =$ _______ + _______

$h^2 =$ _______ + _______

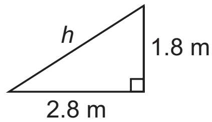
$h^2 =$ _______

$h = \sqrt{\_\_\_\_\_\_\_}$

$h \doteq$ _______

The length of the hypotenuse is about _______ m.

**b)**




$h^2 =$ _______ + _______

$h^2 =$ _______ + _______

$h^2 =$ _______

$h = \sqrt{\_\_\_\_\_\_\_}$

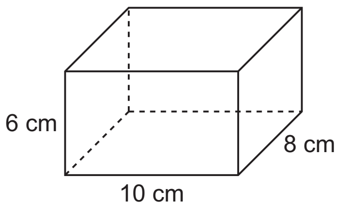
$h \doteq$ _______

The length of the hypotenuse is about _______ m.

# 1.3 Skill Builder

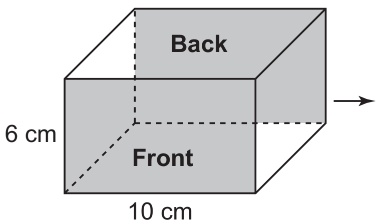
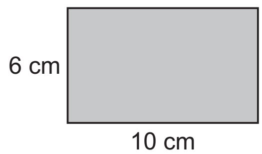
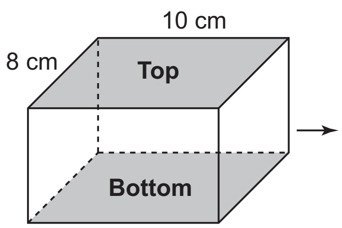
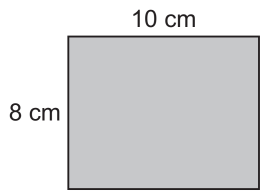
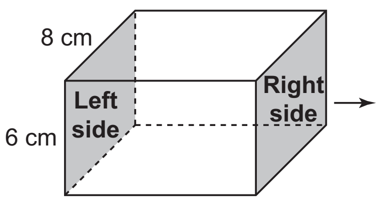
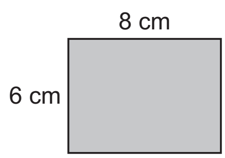
## Surface Areas of Rectangular Prisms

The **surface area** of a rectangular prism is the sum of the areas of its 6 rectangular faces. Look for matching faces with the same areas.




*The matching faces in each pair have the same area. We find the area of one face and multiply by 2.*

For each rectangular face, area equals its length times its width.

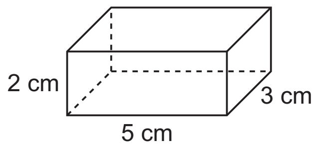
| Matching Faces | Diagram | Corresponding Area (cm$^2$) |
|---|---|---|
|   |   | $2(10 \times 6) = 120$ |
|   |   | $2(10 \times 8) = 160$ |
|   |   | $2(8 \times 6) = 96$ |
| **Total** | | 376 |

The surface area is 376 cm$^2$.

## *Check*

**1.** Determine the surface area of each rectangular prism.

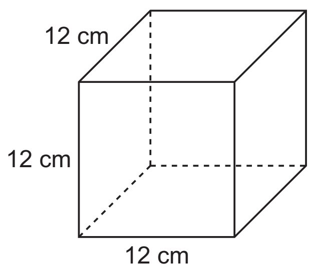
**a)**




| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Front<br>Back | ___ cm<br>___ cm | 2(___ × ___) = ___ |
| Top<br>Bottom | ___ cm<br>___ cm | 2(___ × ___) = ___ |
| Right<br>Left | ___ cm<br>___ cm | 2(___ × ___) = ___ |
| **Total** | | ___ |

The surface area is ___ $cm^2$.

**b)**




| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Front<br>Back | ___ cm<br>___ cm | 2(___ × ___) = ____ |
| Top<br>Bottom | ___ cm<br>___ cm | 2(___ × ___) = ____ |
| Right<br>Left | ___ cm<br>___ cm | 2(___ × ___) = ____ |
| **Total** | | ____ |

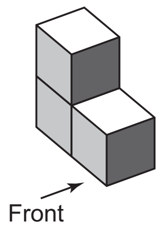
The surface area is ____ $cm^2$.

## 1.3 Surface Areas of Objects Made from Right Rectangular Prisms

**FOCUS** **Find the surface areas of objects made from rectangular prisms.**

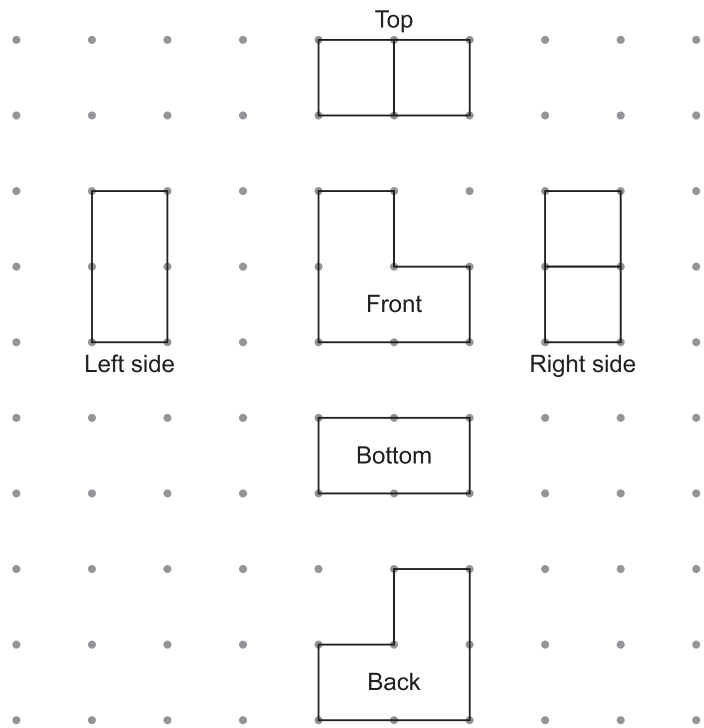
### *Example 1* Finding the Surface Area of an Object Made from Cubes

Make this object with 1-cm cubes.
What is the surface area of the object?




### *Solution*

Think of tracing each face, or "opening" the object.




*Turn the object to see each view.*

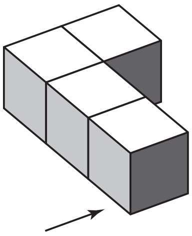
Look for matching views.

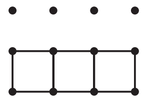
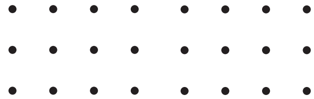
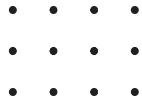
| **Matching Views** | **Corresponding Area ($cm^2$)** |
|---|---|
| Front / Back | $2(3) = 6$ |
| Top / Bottom | $2(2) = 4$ |
| Right / Left | $2(2) = 4$ |
| **Total** | 14 |

The surface area is 14 $cm^2$.

## *Check*

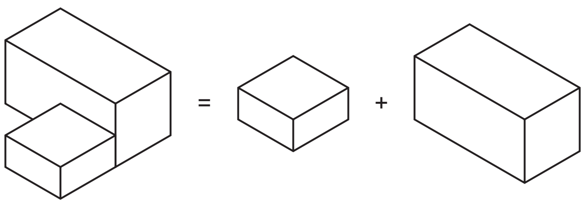
**1.** Make this object with 1-cm cubes, then find its surface area.



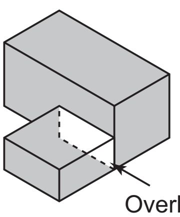

| Matching Views | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Front<br>Back |  | 2(__) = __ |
| Top<br>Bottom |  | 2(__) = __ |
| Right<br>Left |  | 2(__) = __ |
| **Total** | | ___ |

The surface area is ____ $cm^2$.

A **composite object** is made from 2 or more objects.




To find the surface area of a composite object, imagine dipping the object in paint. The surface area is the area of all the faces covered in paint.

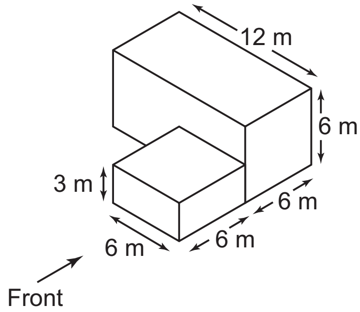



*Where objects overlap, there is a hidden surface. The paint doesn't reach the hidden surface.*

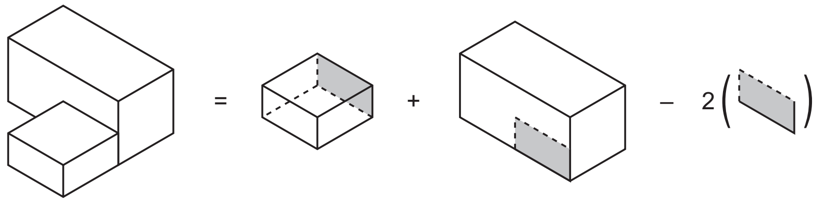
The overlap is not painted, so it is not part of the surface area.

## *Example 2* Finding the Surface Area of a Composite Object

Find the surface area of this composite object.




## *Solution*




Surface area of composite object = Surface area of smaller prism + Surface area of larger prism – 2(Area of overlap)

**Surface area of smaller prism**

| **Matching Faces** | **Diagram** | **Corresponding Area ($m^2$)** |
|---|---|---|
| Front<br>Back<br>Right<br>Left | 6 m<br>3 m | $4(6 \times 3) = 72$ |
| Top<br>Bottom | 6 m<br>6 m | $2(6 \times 6) = 72$ |
| **Total** | | 144 |

The surface area is 144 $m^2$.

**Surface area of larger prism**

| **Matching Faces** | **Diagram** | **Corresponding Area ($m^2$)** |
|---|---|---|
| Front<br>Back<br>Top<br>Bottom | 12 m<br>6 m | $4(12 \times 6) = 288$ |
| Right<br>Left | 6 m<br>6 m | $2(6 \times 6) = 72$ |
| **Total** | | 360 |

The surface area is 360 $m^2$.

**Area of overlap**

| **Diagram** | **Corresponding Area ($m^2$)** |
|---|---|
| 6 m<br>3 m | $6 \times 3 = 18$ |

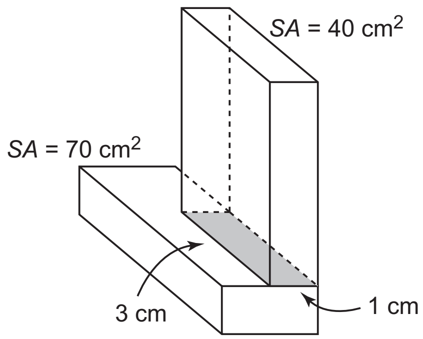
The area of overlap is 18 $m^2$.

SA of composite object $= 144 + 360 - 2(18) = 468$
The surface area of the composite object is 468 $m^2$.

*SA means surface area.*

## *Check*

**1.** The diagram shows the surface areas of the two prisms that make up a composite object.




**a)** What is the area of the overlap?

The overlap is a _____-cm by _____-cm rectangle.

Area of overlap = _____ cm × _____ cm

= _____ $cm^2$

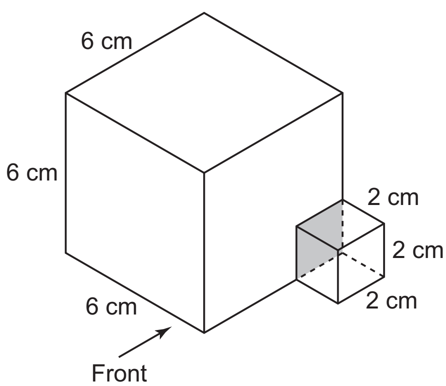
**b)** What is the surface area of the composite object?

SA composite object = SA smaller prism + SA larger prism − 2(Area of overlap)

= _____ $cm^2$ + _____ $cm^2$ − 2(_____) $cm^2$

= _____ $cm^2$

**2.** Find the surface area of this composite object.




A cube has _____ congruent faces.

**Surface area of larger cube**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Front<br>Back<br>Top<br>Bottom<br>Right<br>Left | __ cm<br>__ cm | 6(__ × __) = ____ |
| **Total** | | _____ |

The surface area is ____ $cm^2$.

**Surface area of smaller cube**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Front<br>Back<br>Top<br>Bottom<br>Right<br>Left | __ cm<br>__ cm | 6(__ × __) = ____ |
| **Total** | | _____ |

The surface area is ___ $cm^2$.

**Area of overlap**

| Diagram | Corresponding Area ($cm^2$) |
|---|---|
| __ cm<br>__ cm | __ × __ = __ |

The area of overlap is __ $cm^2$.

SA composite object = SA larger cube + ______________ − ____________________

= _____ + _____ − 2(__)

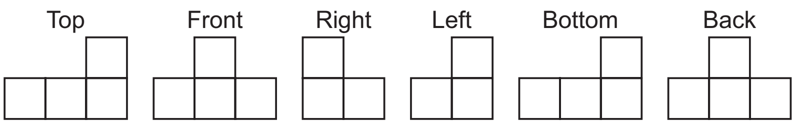
= _____

The surface area of the composite object is _____ $cm^2$.

---

## Practice

**1.** The diagram shows the 6 views of an object made from 1-cm cubes.
Identify pairs of matching views in the first column of the table.
Then, find the surface area of the object.

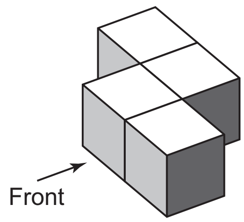



| Matching Views | Corresponding Area ($cm^2$) |
|---|---|
| Front / _______ | _______________ |
| Top / ________ | _______________ |
| Right / _______ | _______________ |
| **Total** | _______________ |

The surface area is ___ $cm^2$.

**2.** Each object is made with 1-cm cubes. Find the surface area of each object.

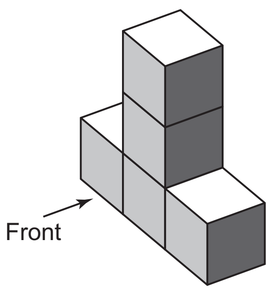
**a)**




| Matching Views | Diagram | Corresponding Area (cm$^2$) |
|---|---|---|
| Front<br>Back | | 2(__) = __ |
| Top<br>Bottom | | 2(__) = __ |
| Right<br>Left | | 2(__) = __ |
| **Total** | | ___ |

The surface area is ___ cm$^2$.

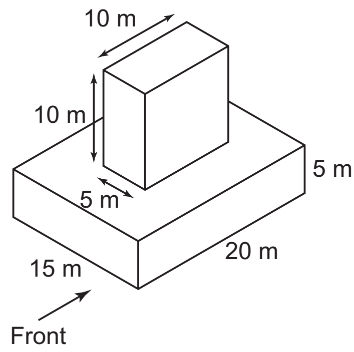
**b)**




| Matching Views | Diagram | Corresponding Area (cm$^2$) |
|---|---|---|
| Front<br>Back | | 2(__) = ___ |
| Top<br>Bottom | | ________________ |
| Right<br>Left | | ________________ |
| **Total** | | ___ |

The surface area is ___ cm$^2$.

**3.** Find the surface area of this composite object.




**Surface area of larger prism**

| Matching Faces | Diagram | Corresponding Area ($m^2$) |
|---|---|---|
| Front<br>Back | | 2(___ × __) = ____ |
| Top<br>Bottom | | ______________ |
| Right<br>Left | | ______________ |
| **Total** | | ______________ |

The surface area is ____ $m^2$.

**Surface area of smaller prism**

| Matching Faces | Diagram | Corresponding Area ($m^2$) |
|---|---|---|
| Front<br>Back | | 2(__ × ___) = ____ |
| Top<br>Bottom | | ______________ |
| Right<br>Left | | ______________ |
| **Total** | | ______________ |

The surface area is ____ $m^2$.

**Area of overlap**

| Diagram | Corresponding Area ($m^2$) |
|---|---|
| | ___ × ___ = ___ |

The area of overlap is ___ $m^2$.

**Surface area of composite object**

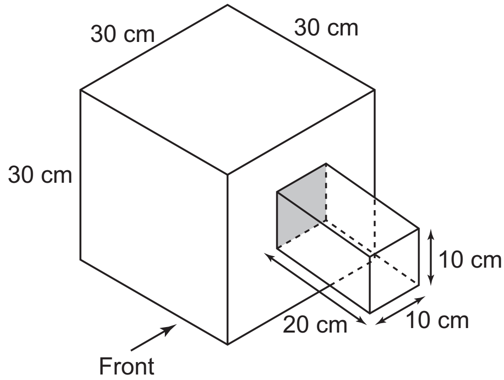
SA composite object = ____________ + ____________ − ____________

= ____ + ____ − 2(___)

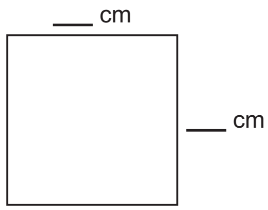
= _____

The surface area of the composite object is _____ $m^2$.

**4.** Find the surface area of this composite object.




**Surface area of cube**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Front / Back<br>Top / Bottom<br>Right / Left |  | 6(___ × ___) = _____ |
| **Total** | | _____ |

The surface area is _____ $cm^2$.

**Surface area of rectangular prism**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Front / Back | | 2(___ × ___) = ____ |
| Top / Bottom | | ________________ |
| Right / Left | | ________________ |
| **Total** | | ________________ |

The surface area is _____ $cm^2$.

**Area of overlap**

| Diagram | Corresponding Area ($cm^2$) |
|---|---|
| | ___ × ___ = ____ |

The area of overlap is ____ $cm^2$.

**Surface area of composite object**

SA composite object = ________________ + ________________ − ________________

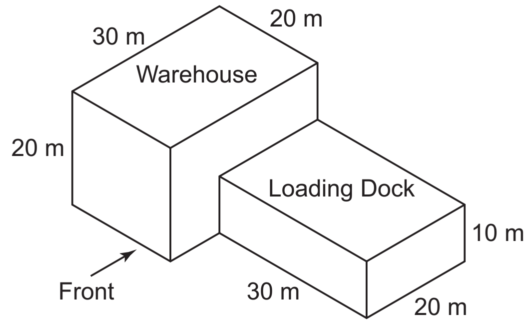
= _____ + _____ − ______

= ______________________

The surface area of the composite object is _____ $cm^2$.

**5.** A loading dock is attached to one wall of a warehouse. The exterior of the buildings is to be painted at a cost of $2.50/m$^2$. How much will it cost to paint the buildings?

Will the bottom of the warehouse and loading dock be painted? ________




**Surface area of warehouse to be painted**

| Matching Faces | Diagram | Corresponding Area (m$^2$) |
|---|---|---|
| Front<br>Back | | 2(___ × ___)<br>= ____ |
| Top<br>Sides | | 3(___ × ___)<br>= ____ |
| **Total** | | _________ |

The surface area of the warehouse to be painted is _____ m$^2$.

**Area of overlap**

| Diagram | Corresponding Area (m$^2$) |
|---|---|
| | ___ × ___ = ____ |

The area of overlap is ____ m$^2$.

**Surface area of loading dock to be painted**

| Matching Faces | Diagram | Corresponding Area (m$^2$) |
|---|---|---|
| Front<br>Back | | 2(___ × ___)<br>= ____ |
| Top | | ___ × ___<br>= ____ |
| Sides | | 2(___ × ___)<br>= ____ |
| **Total** | | _________ |

The surface area of the loading dock to be painted is _____ m$^2$.

Surface area of composite object to be painted

_____ + _____ − _______ = _____

The surface area of the composite object to be painted is _____ m$^2$.

So, the area to be painted is _____ m$^2$.

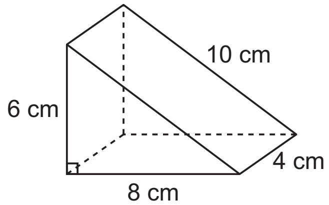
The cost per square metre is: $_____

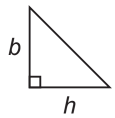
The cost to paint the buildings is: _____ × $_____ = _____

# 1.4 Skill Builder

## Surface Areas of Triangular Prisms

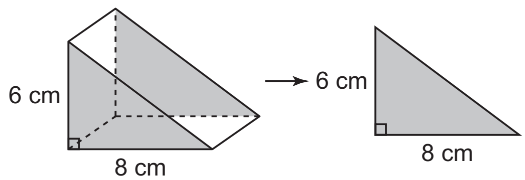
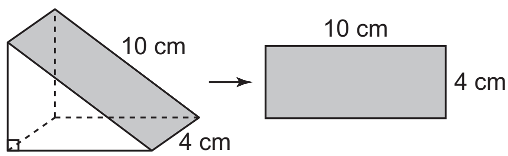
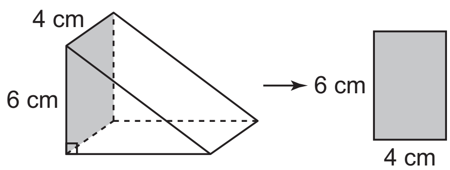
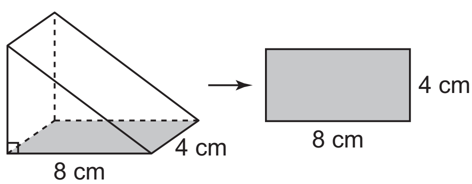
To find the surface area of a right triangular prism, add the areas of its 5 faces.
Look for matching faces with the same areas.




$A = \frac{1}{2}bh$

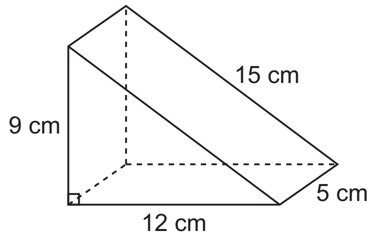
*There are 2 congruent triangular faces. Find the area of one, then multiply it by 2.*

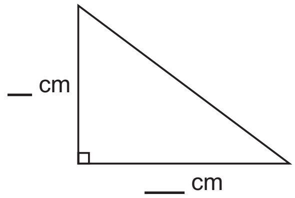
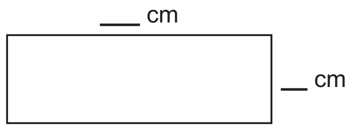
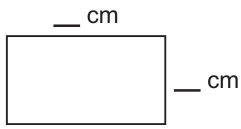
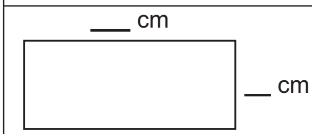
| Matching Faces | Diagram | Corresponding Area (cm$^2$) |
|---|---|---|
| Triangular |  | $2(\frac{1}{2} \times 6 \times 8) = 48$ |
| Rectangular |  | $10 \times 4 = 40$ |
| |  | $6 \times 4 = 24$ |
| |  | $8 \times 4 = 32$ |
| **Total** | | 144 |

The surface area is 144 cm$^2$.

## *Check*

**1.** Find the surface area of the triangular prism.




| **Matching Faces** | **Diagram** | **Corresponding Area ($cm^2$)** |
|---|---|---|
| Triangular |   | $2(\frac{1}{2} \times \_\_ \times \_\_) = \_\_$ |
| Rectangular |   | $\_\_ \times \_\_ = \_\_$ |
| |   | $\_\_ \times \_\_ = \_\_$ |
| |   | $\_\_ \times \_\_ = \_\_$ |
| **Total** | | ___ |

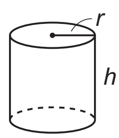
The surface area is ____ $cm^2$.

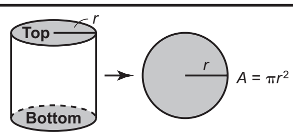
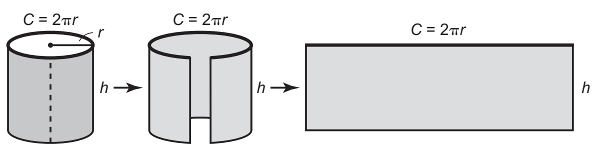
## Surface Areas of Cylinders

To find the surface area of a right cylinder, add the areas of:

- the 2 circular faces
- the curved surface

Look for matching faces with the same areas.

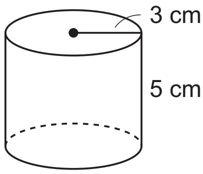


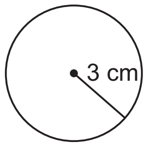
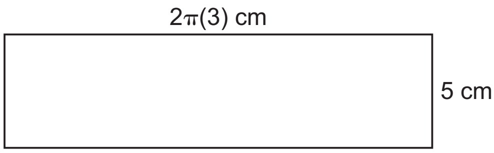

| **Matching Faces** | **Diagram** | **Corresponding Area** |
|---|---|---|
| Top<br>Bottom |   | $2 \times \pi r^2$ |
| Curved surface |   | $2\pi rh$ |

*The side can be unrolled into a rectangle, whose length is the circumference of the circle.*

The surface area is: $2\pi r^2 + 2\pi rh$

To calculate the surface area of this cylinder:



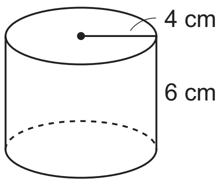

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Top<br>Bottom |   | $2 \times \pi \times 3^2$<br>$\doteq 56.55$ |
| Curved surface |   | $2 \times \pi \times 3 \times 5$<br>$\doteq 94.25$ |
| **Total** | | 150.80 |

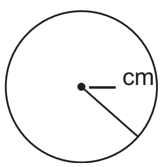
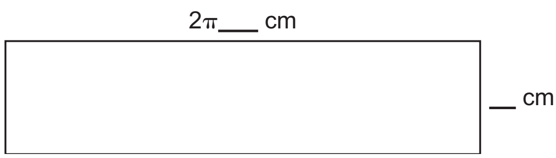
*The dimensions of the cylinder are given to the nearest centimetre, so we give the surface area to the nearest square centimetre.*

The surface area is about 151 $cm^2$.

## *Check*

**1.** Find the surface area of the cylinder.




| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Top<br>Bottom |   | ___ × ___ × ___<br>≐ _______ |
| Curved surface |   | ___ × ___ × ___ × ___<br>≐ _______ |
| **Total** | | _______ |

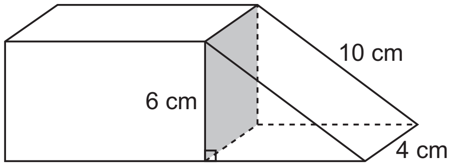
The surface area is about ____ $cm^2$.

# 1.4 Surface Areas of Other Composite Objects

**FOCUS** **Find the surface areas of composite objects made from right prisms and right cylinders.**

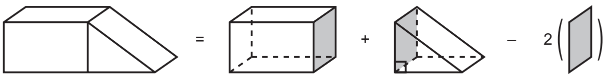
## *Example 1* Finding the Surface Area of a Composite Object Made from a Rectangular Prism and a Triangular Prism

Find the surface area of this composite object.

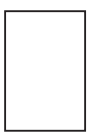



*The shaded area is the area of overlap.*

### *Solution*

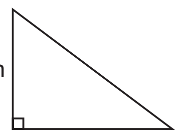
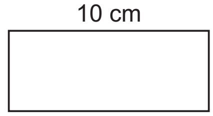
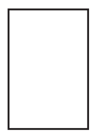
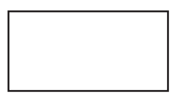



**Surface area of rectangular prism**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Front<br>Back |  6 cm, 10 cm | $2(6 \times 10)$<br>$= 120$ |
| Top<br>Bottom |  10 cm, 4 cm | $2(10 \times 4)$<br>$= 80$ |
| Right<br>Left |  6 cm, 4 cm | $2(6 \times 4)$<br>$= 48$ |
| **Total** | | 248 |

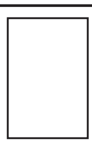
The surface area is 248 $cm^2$.

**Surface area of triangular prism**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Triangular |  6 cm, 8 cm | $2(\frac{1}{2} \times 6 \times 8)$<br>$= 48$ |
| Rectangular |  10 cm, 4 cm | $10 \times 4 = 40$ |
| |  6 cm, 4 cm | $6 \times 4 = 24$ |
| |  4 cm, 8 cm | $8 \times 4 = 32$ |
| **Total** | | 144 |

The surface area is 144 $cm^2$.

**Area of overlap**

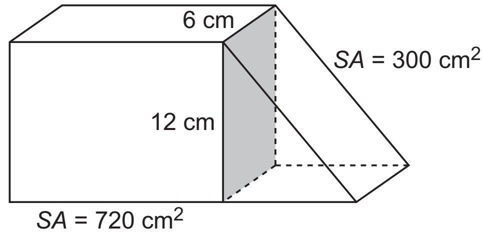
| Diagram | Corresponding Area ($cm^2$) |
|---|---|
|  6 cm, 4 cm | $6 \times 4 = 24$ |

The area of overlap is 24 $cm^2$.

Surface area of composite object $= 248 + 144 - 2(24) = 344$
The surface area of the composite object is 344 $cm^2$.

## *Check*

**1.** The diagram shows the surface area of the two prisms that make up a composite object.




**a)** What is the area of the overlap?

The overlap is a ____-cm by ____-cm rectangle.

Area of overlap = ____ cm × ____ cm = ____ $cm^2$

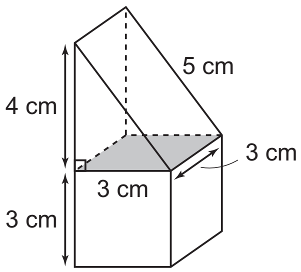
**b)** What is the surface area of the composite object?

Surface area of composite object = Surface area of 2 prisms − 2(Area of overlap)

= _____ + _____ − _____ = _____

The surface area of the composite object is __________.

**2.** Find the surface area of this composite object.




**Surface area of triangular prism**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Triangular | __ cm, __ cm | 2(__ × ____ × ____) = ____ |
| Rectangular | __ cm, __ cm | ____ × ____ = ____ |
| | __ cm, __ cm | ____ × ____ = ____ |
| | __ cm, __ cm | ____ × ____ = ____ |
| **Total** | | _____ |

The surface area is _____ $cm^2$.

**Surface area of cube**

| Matching Faces | Diagram | Corresponding Area (cm$^2$) |
|---|---|---|
| Front<br>Back<br>Top<br>Bottom<br>Right<br>Left | __ cm<br>__ cm | 6(____ × ____) = ____ |
| **Total** | | ____ |

The surface area is ____ cm$^2$.

**Area of overlap**

| Diagram | Corresponding Area (cm$^2$) |
|---|---|
| __ cm<br>__ cm | ___ × ___ = ___ |

The area of overlap is _____ cm$^2$.

Surface area of composite object = Surface area of 2 prisms − 2(Area of overlap)

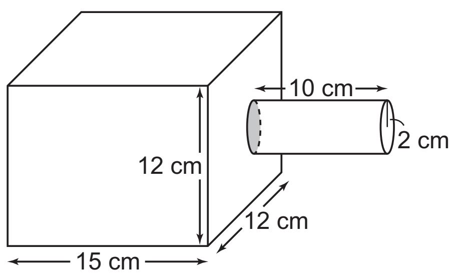
= ______ + ______ − ______

= ______

The surface area of the composite object is ______ cm$^2$.

## *Example 2* Finding the Surface Area of a Composite Object Made from a Rectangular Prism and a Cylinder

Find the surface area of this object.




**Surface area of rectangular prism**

| Matching Faces | Diagram | Corresponding Area (cm$^2$) |
|---|---|---|
| Front<br>Back<br>Top<br>Bottom | 15 cm<br>12 cm | 4(12 × 15) = 720 |
| Right<br>Left | 12 cm<br>12 cm | 2(12 × 12) = 288 |
| **Total** | | 1008 |

The surface area is 1008 cm$^2$.

**Surface area of cylinder**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Top<br>Bottom |  | $2 \times \pi \times 2^2 \doteq 25.13$ |
| Curved surface |  | $2 \times \pi \times 2 \times 10 \doteq 125.67$ |
| **Total** | | 150.80 |

The surface area is about 150.80 $cm^2$.

**Area of overlap**

| Diagram | Corresponding Area ($cm^2$) |
|---|---|
|  | $\pi \times 2^2 \doteq 12.57$ |

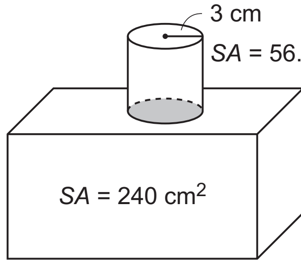
The area of overlap is about 12.57 $cm^2$.

$$\begin{aligned}\text{SA composite object} &= \text{SA rectangular prism} + \text{SA cylinder} - 2(\text{Area of overlap})\\ &\doteq 1008 + 150.80 - 2(12.57)\\ &\doteq 1133.66\end{aligned}$$

The surface area is about 1134 $cm^2$.

## *Check*

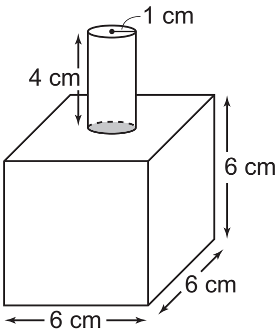
**1.** The diagram shows the surface area of the rectangular prism and cylinder that make up a composite object.



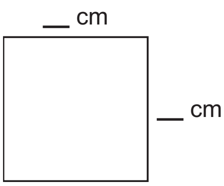

**a)** What is the area of the overlap?
The overlap is a ________________.
Area of overlap = ______________
$\doteq$ ______________ $cm^2$

**b)** What is the surface area of the composite object?
SA composite object = SA ________ + SA ________ − 2(________________)
= ________ + ________ − __________
= ________
The surface area of the composite object is about ________ $cm^2$.

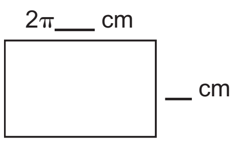
**2.** Find the surface area of this composite object.




**Surface area of cube**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Front<br>Back<br>Top<br>Bottom<br>Right<br>Left |  | 6(____ × ____) = ____ |
| **Total** | | ______ |

**Surface area of cylinder**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Top<br>Bottom |  | ____ × ____ × ____ ≐ ______ |
| Curved surface |  | ____ × ____ × ____ × ____ ≐ ______ |
| **Total** | | ______ |

**Area of overlap**

| Diagram | Corresponding Area ($cm^2$) |
|---|---|
|  | _____ × _____ ≐ _____ |

SA composite object = SA _____ + SA _________ − 2(______________)

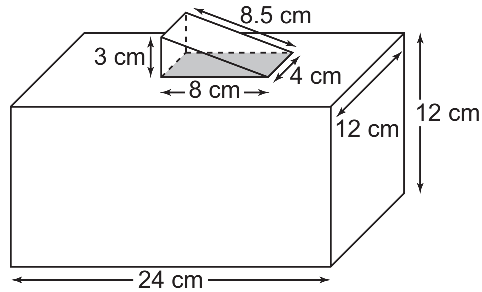
≐ ________ + ________ − ________

≐ ________

The surface area of the composite object is about _____ $cm^2$.

## Practice

**1.** Find the surface area of this composite object.




**Surface area of rectangular prism**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Front<br>Back<br>Top<br>Bottom | | ________ = ________ |
| Right<br>Left | | ________ = ______ |
| **Total** | | ______ |

The surface area is ______ $cm^2$.

**Surface area of triangular prism**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Triangular | | ____________ |
| Rectangular | | ____________ |
| | | ____________ |
| | | ____________ |
| **Total** | | ________ |

The surface area is ______ $cm^2$.

**Area of overlap**

| Diagram | Area ($cm^2$) |
|---|---|
| | __ × __ = __ |

The area of overlap is _____ $cm^2$.

**Surface area of composite object**

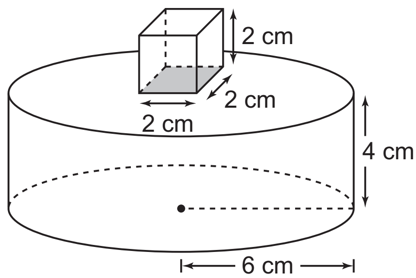
SA composite object

= ____________________

= ____________________

The surface area of the composite object is _______ $cm^2$.

**2.** Find the surface area of this composite object.




**Surface area of cube**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Front<br>Back<br>Top<br>Bottom | | 6(____ × ____) = ____ |
| **Total** | | ____ |

The surface area is ____ $cm^2$.

**Surface area of cylinder**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Top<br>Bottom | | ____ × ____ × ___ ≐ ______ |
| Curved surface | | ____ × ____ × ____ × ____ ≐ _______ |
| **Total** | | ________ |

The surface area is about ________ $cm^2$.

**Area of overlap**

| Diagram | Corresponding Area ($cm^2$) |
|---|---|
| | ___ × ___ = ___ |

The area of overlap is ___ $cm^2$.

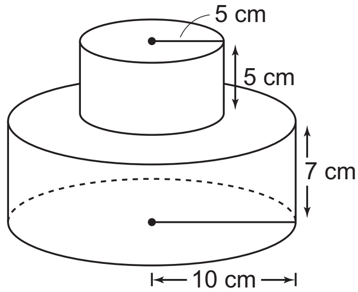
**Surface area of composite object**

SA composite object ≐ ________ + ________ − ________

≐ ________

The surface area of the composite object is about ________ $cm^2$.

**3.** Calculate the surface area of the cake at the right. Write your answer to the nearest tenth.




**Surface area of smaller cake**

| **Matching Faces** | **Diagram** | **Corresponding Area ($cm^2$)** |
|---|---|---|
| Top<br>Bottom | | ____ × ____ × ____ ≐ ________ |
| Curved surface | | ____ × ____ × ____ × ____ ≐ ________ |
| **Total** | | ________ |

The surface area is about _________ $cm^2$.

**Surface area of larger cake**

| **Matching Faces** | **Diagram** | **Corresponding Area ($cm^2$)** |
|---|---|---|
| Top Bottom | | ___ × ___ × _____ ≐ _________ |
| Curved surface | | ___ × ___ × ___ × ___ ≐ _______ |
| **Total** | | ________ |

The surface area is about _________ $cm^2$.

**Area of overlap**

| **Diagram** | **Corresponding Area ($cm^2$)** |
|---|---|
| | ___ × ___ ≐ ______ |

The area of overlap is about _____ $cm^2$.

Surface area of cake ≐ _________ + _________ − _________
≐ _________

The surface area of the cake is about _________ $cm^2$.

# Unit 1 Puzzle

**Square and Square-Root Days**

A date in a given year can be written as the month number followed by the day number. For example, October 25 can be written as 10/25.

- In a *square-root day*, the month is the square root of the day.
  For example, March 9 is a square-root day because it is written as 3/9, and 3 is the square root of 9.

  List all the square-root days in a year.

  ______________________________

- In a *square day*, the month is the square of the day.
  For example, April 2 is a square day because it is written as 4/2, and 4 is the square of 2.

  List all the square days in a year.

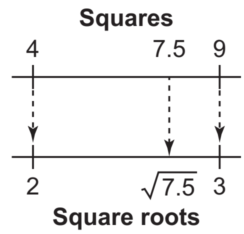
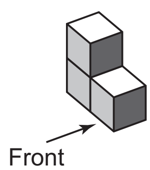
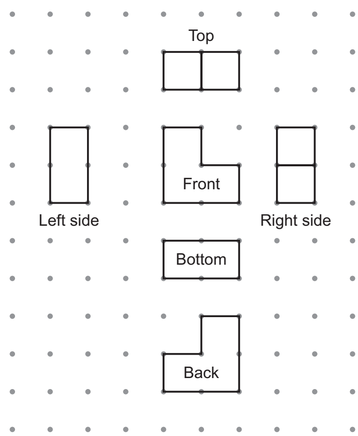
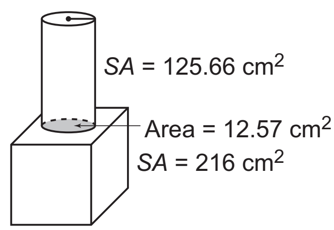
  ______________________________

- A *square year* is a year which is a perfect square.
  For example, the year 1600 is a square year because $1600 = 40 \times 40$.

  List all the square years from 1000 to the present.

  ______________________________

# Unit 1 Study Guide

| Skill | Description | Example |
|---|---|---|
| Identify fractions that are perfect squares and find their square roots. | A fraction is a perfect square if it can be written as the product of 2 equal fractions. The square root is one of the 2 equal fractions. | $\frac{16}{25} = \frac{4}{5} \times \frac{4}{5}$<br>$\sqrt{\frac{16}{25}} = \frac{4}{5}$ |
| Identify decimals that are perfect squares. | Use a calculator. The square root is a repeating or terminating decimal. | $\sqrt{1.69} = 1.3$ |
| Estimate square roots of numbers that are not perfect squares. | Find perfect squares close to the number.<br><br>Use the squares and square roots number lines. | $\sqrt{\frac{3}{10}} \doteq \sqrt{\frac{4}{9}} \doteq \frac{2}{3}$ 3 is close to 4; 10 is close to 9.<br> |
| Calculate the surface area of a composite object. | Add the areas of each of the 6 views.<br><br>Or<br>Add surface areas of the parts, then subtract for the overlap. | <br><br>The surface area is 14 square units.<br><br>$SA = 216 + 125.66 - 2(12.57)$<br>$= 316.52$<br>The surface area is about 317 cm$^2$. |

# Unit 1 Review

1.1 **1.** Calculate the number whose square root is:

**a)** $\frac{3}{7}$

$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

$\frac{3}{7}$ is a square root of ___.

**b)** 9.9

$9.9 \times 9.9 =$ ______

9.9 is a square root of ______.

**2.** Complete the table.

| | Fraction | Is numerator a perfect square? | Is denominator a perfect square? | Is fraction a perfect square? |
|---|---|---|---|---|
| **a)** | $\frac{25}{81}$ | ____________ | ____________ | _____ |
| **b)** | $\frac{7}{4}$ | ____________ | ____________ | _____ |
| **c)** | $\frac{49}{65}$ | ____________ | ____________ | _____ |

**3.** Complete the table.

| | Decimal | Value of Square Root | Type of Decimal | Is decimal a perfect square? |
|---|---|---|---|---|
| **a)** | 5.29 | ____________ | ________________________ ________________________ | _____ |
| **b)** | 156.25 | ____________ | ________________________ ________________________ | _____ |
| **c)** | 6.4 | ____________ | ________________________ ________________________ | _____ |

**4.** Find the square root of each number.

**a)** $\sqrt{\frac{25}{81}} = \frac{\_\_}{\_\_}$

**b)** $\sqrt{59.29} =$ _____

1.2 **5.** Estimate $\sqrt{14.5}$. Explain your estimate.

14.5 is between ____ and ____.

So, $\sqrt{14.5}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$. That is, $\sqrt{14.5}$ is between ____ and ____.

Since 14.5 is closer to ____ than ____, $\sqrt{14.5}$ is closer to ____ than ____.

So, $\sqrt{14.5}$ is between ____ and ____, and closer to ____.

**6.** Estimate each square root. Explain.

**a)** $\sqrt{\frac{2}{13}}$

2 is close to ____; 13 is close to ____.

So, $\sqrt{\frac{2}{13}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\doteq \frac{\_\_\_\_}{\_\_\_\_}$

**b)** $\sqrt{\frac{11}{70}}$

11 is close to ____; 70 is close to ____.

So, $\sqrt{\frac{11}{70}} \doteq \sqrt{\frac{\_\_\_\_}{\_\_\_\_}}$

$\doteq \frac{\_\_\_\_}{\_\_\_\_}$

**7.** Identify a decimal that has a square root between the two given numbers. Check the answer.

**a)** 2 and 3

$2^2 =$ ____ and $3^2 =$ ____

So, any number between ____ and ____ has a square root between 2 and 3.

Choose ____.

Check: $\sqrt{\_\_\_\_} \doteq$ ____

The decimal ____ is one possible answer.

**b)** 6.5 and 7.5

______ = ______ and ______ = ______

So, any number between ______ and ______ has a square root between 6.5 and 7.5.

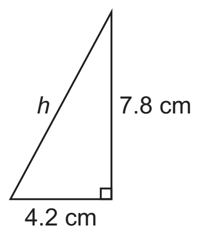
Choose ______.

Check: $\sqrt{\_\_\_\_\_\_} \doteq$ ______.

The decimal ______ is one possible answer.

**8.** Find the length of the hypotenuse of each right triangle.

**a)**




$h^2 =$ ______ + ______

$h^2 =$ ______ + ______

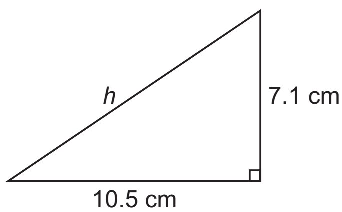
$h^2 =$ ______

$h = \sqrt{\_\_\_\_\_\_}$

$h \doteq$ ______

The length of the hypotenuse is about ______ cm.

**b)**




$h^2 =$ ______ + ______

$h^2 =$ ______ + ______

$h^2 =$ ______

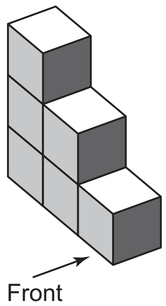
$h = \sqrt{\_\_\_\_\_\_}$

$h \doteq$ ______

________________________________

________________________________

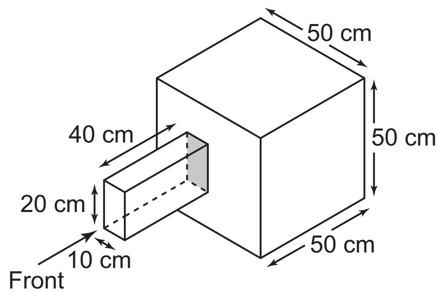
**1.3** **9.** This object is made from 1-cm cubes. Find its surface area.




| Matching Views | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| ________<br>________ | | ________ |
| ________<br>________ | | ________ |
| ________<br>________ | | ________ |
| **Total** | | ____ |

The surface area is _____ $cm^2$.

**10.** Calculate the surface area of this composite object.




**Surface area of cube**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| ______ /______<br>____ /________<br>______/______ | | 6(___ × ___) = _______ |
| **Total** | | _______ |

The surface area is _______ $cm^2$.

**Surface area of rectangular prism**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| ______ /______ | | ____________________ |
| ____ /________ | | ____________________ |
| ______/______ | | ____________________ |
| **Total** | | ____________________ |

The surface area is ______ $cm^2$.

**Area of overlap**

| Diagram | Corresponding Area ($cm^2$) |
|---|---|
| | ____ × ____ = ____ |

The area of overlap is ____ $cm^2$.

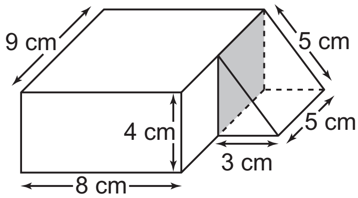
SA composite object = ________________ + _________________ − ___________________

= _______ + _______ − _______

= _______

The surface area of the composite object is _______ $cm^2$.

**1.4** **11.** Find the surface area of this composite object.




**Surface area of rectangular prism**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| ________<br>________ | | ______________ |
| ________<br>________ | | ______________ |
| ________<br>________ | | ______________ |
| **Total** | | _______ |

The surface area is _____ $cm^2$.

**Area of overlap**

| Diagram | Corresponding Area ($cm^2$) |
|---|---|
| | ___________ |

The area of overlap is ____ $cm^2$.

**Surface area of triangular prism**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Triangular | | _____________<br>_____________ |
| Rectangular | | _____________ |
| | | _____________ |
| | | _____________ |
| **Total** | | ____ |

The surface area is ____ $cm^2$.

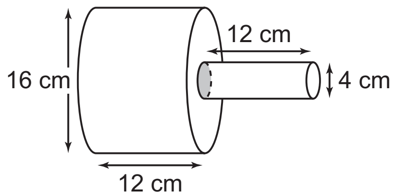
SA = ______________________ + _____________________ − ____________________

= _____ + ____ − ______

= _____

The surface area of the composite object is _____ $cm^2$.

**12.** Find the surface area of this composite object.




The larger cylinder has diameter ____ cm, so its radius is ___ cm.
The smaller cylinder has diameter ___ cm, so its radius is ___ cm.

**Surface area of smaller cylinder**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Top<br>Bottom | | __ × __ × __ ≐ ______ |
| Curved surface | | __ × __ × __ × __ ≐ ________ |
| **Total** | | ________ |

The surface area is about ________ $cm^2$.

**Surface area of larger cylinder**

| Matching Faces | Diagram | Corresponding Area ($cm^2$) |
|---|---|---|
| Top<br>Bottom | | __ × __ × __ ≐ ________ |
| Curved surface | | __ × __ × __ × __ ≐ ________ |
| **Total** | | _________ |

The surface area is about _________ $cm^2$.

**Area of overlap**

| Diagram | Corresponding Area ($cm^2$) |
|---|---|
| | __ × __ ≐ _______ |

The area of overlap is about _______ $cm^2$.

Surface area of the composite object ≐ _______ + _________ − _________
≐ _________

The surface area is about ________ $cm^2$.