UNIT 4

Linear Relations

What You'll Learn

- Use expressions and equations to write patterns and work with them.
- Use substitution to work with patterns to find more information.
- Graph and analyze linear relations.
- Use interpolation and extrapolation to gather more information from graphs.

Why It's Important

Patterns and linear relations are used by

- book printers, to quote the cost of a job
- managers, to plan for new hiring

Key Words

variable
expression
equation
table of values
relation
linear relation
coordinates
coordinate grid
discrete
origin
vertical
horizontal
oblique
interpolation
extrapolation

4.1 Skill Builder

Algebraic Expressions

We can use a variable, such as n, to represent a number in an expression.

For example:

- 4 more than a number: $4 + n$, or $n + 4$
- 5 times a number: $5n$
- 2 less than a number: $n - 2$
- A number divided by 10: $\frac{n}{10}$

A variable is a letter, and it is always written in italics.

To evaluate an expression, we replace a variable with a number.

Check

1. Find the value of each expression when $x = 3$.

a) $3x + 5$
3(____) + 5 = ______ + 5
= ______

b) $6 + 8x$
6 + 8(____) = 6 + ______
= ______

2. Find the value of each expression when $n = 8$.

a) $8n - 4$
8(____) − ____ = ____ − ____
= ____

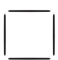
b) $20 - 2n$
____ − 2(____) = ____ − ____
= ____

Relationships in Patterns

Here is a pattern made with toothpicks.

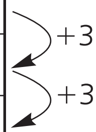
Figure 1 Figure 2 Figure 3

A pattern rule for the number of toothpicks is:

Start with . Add  each time.
There is a pattern in the numbers as well.
Start with 4. Add 3 each time.

We can also show the pattern using a table of values.

Figure Number	Number of Toothpicks
1	4
2	7
3	10



To extend the pattern, continue to add 3 each time:


Figure 4 Figure 5

Check

1. a) Draw the next 2 figures to extend each pattern.

i)

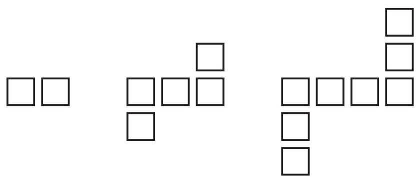


Figure 1 Figure 2 Figure 3 ______ Figure 4 ______ Figure 5

ii)

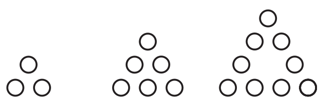


Figure 1 Figure 2 Figure 3 ______ Figure 4 ______ Figure 5

b) Complete each table of values to show each number pattern above.

i)

Figure Number	Number of Squares
1	2
2	______
3	______
______	______
______	______

ii)

Figure Number	Number of Dots
1	3
2	______
3	______
______	______
______	______

4.1 Writing Equations To Describe Patterns

FOCUS **Use equations to describe and solve problems involving patterns.**

Example 1 Writing an Equation to Represent a Written Pattern

Here is a pattern made with sticks.

Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

a) Write an equation that relates the number of sticks to the figure number.

b) What is the number of sticks in the 10th figure?

Solution

a) Record the number of sticks in each figure in a table.

As the figure number increases by 1, the number of sticks increases by 2. Repeated addition of 2 is the same as multiplication by 2.

So, the equation $n = 2f$ may represent the relationship.

Figure Number, f	Number of Sticks, n
1	3
2	5
3	7
4	9
5	11

Check whether the equation is correct.
When $f = 1$, $n = 2(1) = 2$
This is 1 less than 3.
So, add 1.
$2(1) + 1 = 3$

So, an equation is: $n = 2f + 1$

Figure Number, f	Number of Sticks, n
1	$2(1) + 1 = 3$
2	$2(2) + 1 = 5$
3	$2(3) + 1 = 7$
4	$2(4) + 1 = 9$
5	$2(5) + 1 = 11$

b) To find the number of sticks in the 10th figure, substitute $f = 10$ in the equation:

$$\begin{aligned} n &= 2f + 1 \\ &= 2(10) + 1 \\ &= 20 + 1 \\ &= 21 \end{aligned}$$

There are 21 sticks in the 10th figure.

$2f + 1$ represents the number of sticks for any figure number f.

Check

1. For the table below:

Number of Swings, *s*	Number of Hits, *h*
1	5
2	9
3	13
4	17

a) Describe the number of hits in terms of the number of swings.
The number of hits is _____ times the number of swings, plus _____.

b) Write an equation to describe the relationship.
$h =$ _____$s +$ _____

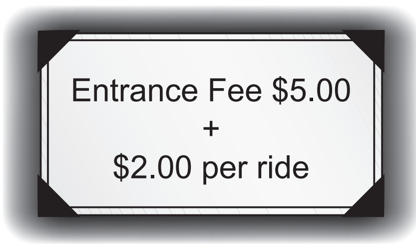
c) Use your equation to find h when $s = 10$.
$h =$ ____________________

____

____

Example 2 Writing an Equation to Represent a Situation

Teagan goes to a carnival. The cost for a ride is shown on a poster at the entrance.



a) Write an equation that relates the total cost, C dollars, to the number of rides, r.

b) Teagan goes on 4 rides. What is his total cost?

Solution

a) The cost is \$5.00, plus \$2.00 per ride.
That is, the cost is: $5.00 + 2.00 \times$ (number of rides)
An equation is: $C = 5.00 + 2.00r$

b) Use the equation: $C = 5.00 + 2.00r$
Substitute: $r = 4$
$C = 5.00 + 2.00(4)$
$= 5.00 + 8.00$
$= 13.00$
Teagan's total cost is \$13.00.

Check

1. Marcel takes a summer job at a book packaging plant.
He gets paid \$50 a day, plus \$2 for every box packed.

a) Write an equation that relates the number of boxes packed to Marcel's pay for a day.
Let P represent his pay for one day, and let b represent the number of boxes packed.
$P =$ ______________________

b) Marcel packed 20 boxes one day. How much did he get paid?
$P =$ ______________________
$=$ ______________________
$=$ ______________________
Marcel got paid ______.

Practice

1. In each equation, find the value of T when $n = 6$.

a) $T = 8 + n$
$T = 8 +$ _____
$=$ __________

b) $T = 3n - 2$
$T = 3$_____ $- 2$
$=$ __________
$=$ __________

c) $T = 12n + 9$
$T =$ __________
$=$ __________
$=$ __________

d) $T = 7n + 3$
$T =$ __________
$=$ __________
$=$ __________

2. a) This pattern of dots continues. Draw the next 2 figures in the pattern.

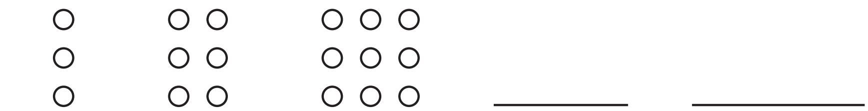


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

b) The pattern is represented in a table of values. Which expression below represents the number of dots in terms of the figure number?

i) $2f$
ii) $3f$
iii) $-3f$
iv) $3f + 1$

____

Figure Number, *f*	Number of Dots, *n*
1	3
2	6
3	9
4	12
5	15

3. a) Look at the pattern of tiles below.
Draw the next 2 figures in the pattern.

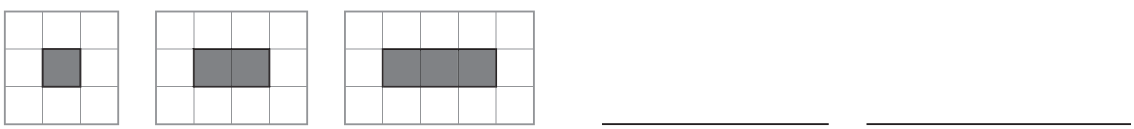


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

b) Complete the table below.

Number of Shaded Tiles, *s*	Number of White Tiles, *w*
1	8
2	_____
_____	_____
_____	_____
_____	_____

c) Write an equation for the number of white tiles, *w*, in terms of the number of shaded tiles, *s*.

$w =$ ____$s +$ ____

d) Use your equation to find *w* when $s = 25$.

$w =$ ______________

$=$ ______________

$=$ ______________

When the number of shaded tiles is 25, there are _____ white tiles.

4. Anabelle is part of the yearbook committee. This year, the set-up cost to print yearbooks is $400, plus $3 for each yearbook printed.

a) Write an equation for the total cost in terms of the number of yearbooks printed.
Let *C* represent the total cost, and let *n* represent the number of yearbooks.

$C =$ _______ $+$ _______

b) Anabelle takes 200 orders for yearbooks this year.
What is the total cost to the yearbook committee?

$C =$ ___________________

$=$ ___________________

$=$ ___________________

The total cost is $ ________.

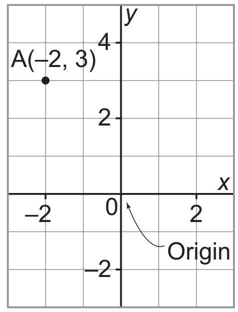
4.2 Skill Builder

The Coordinate Grid

Plot A(−2, 3) on a coordinate grid.
The first number in the ordered pair tells how far you move left or right on the horizontal axis.
The second number tells how far you move up or down on the vertical axis.

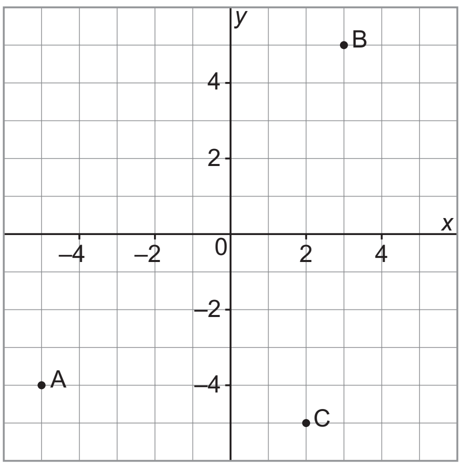
So, to plot A(−2, 3):
Move 2 squares left of the origin,
then move 3 squares up.

−2 is the x-coordinate. 3 is the y-coordinate.



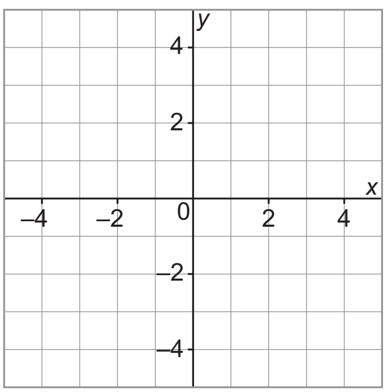
Check

1. What are the coordinates of points A, B, and C?



A(__________)
B(__________)
C(__________)

2. Graph these points on the coordinate grid.
A(−3, 0)   B(2, 4)   C(0, −3)



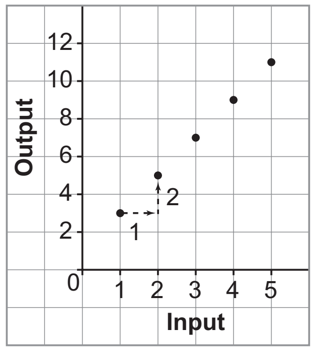
Graphing Relations

This table of values shows how $2n + 1$ relates to n.

	Input, n	Output, $2n + 1$	
+1	1	3	+2
+1	2	5	+2
+1	3	7	+2
+1	4	9	+2
	5	11	

The change in input is constant. The change in output is constant.

Graph of $2n + 1$ against n



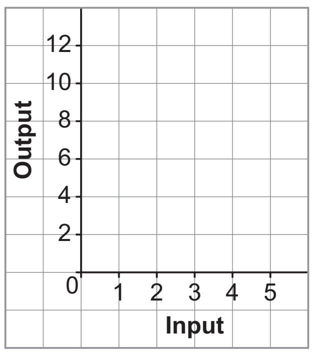
Use the data in the table to graph the relation.

The points lie on a straight line. This is a **linear relation.**
The graph also shows how $2n + 1$ relates to n.
On the graph, we see that each time the input increases by 1, the output increases by 2.

Check

1. a) Graph the data in this table of values.

Input	Output
0	2
1	5
2	8
3	11



b) Is this a linear relation? Explain.

____

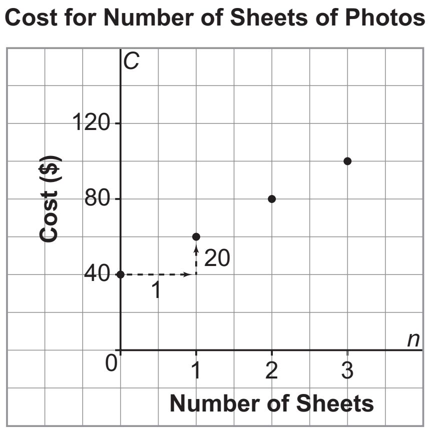
____

4.2 Linear Relations

FOCUS **Analyze the graph of a linear relation.**

A photographer charges $40 for a sitting fee, plus $20 per sheet of prints.
The charges are shown in the table of values and in the graph.

Number of Sheets, *n*	Cost, *C* ($)
0	40
1	60
2	80
3	100



We cannot order part of a sheet of prints.
So, the points in the graph are not joined with a line.
We say that the data are **discrete.**

For different values of *n*, we get different values of *C*.
So the variable *C* *depends* on the value of the variable *n*.
When two variables are related in this way, they form a **relation.**

Linear Relation
When the graph of a relation is a straight line, it is called a **linear relation.**

Example 1 Graphing a Linear Relation from a Table of Values

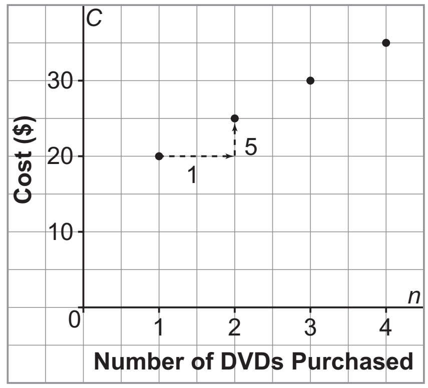
A popular DVD club allows members to purchase DVDs at a reduced price according to the table of values.

a) Graph the data.
b) Should the points be joined? Why or why not?
c) Is the relation linear? Explain.
d) Describe the patterns in the table. How are these patterns shown on the graph?

Number of DVDs Purchased, *n*	Cost, *C* ($)
1	20
2	25
3	30
4	35

Solution

a) Plot the points on a grid.



b) The points should not be joined because you cannot buy part of a DVD.

c) The points on the graph lie on a straight line, so this is a linear relation.

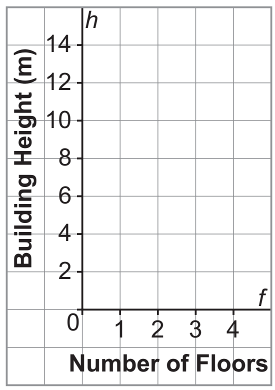
d) The table of values shows that:
The number of DVDs purchased increases by 1 each time.
The cost increases by $5 each time.

To get from one point to the next in the graph, move 1 unit right and 5 units up.

Check

1. a) Graph the data from the table of values.

Number of Floors, *f*	Building Height, *h* (m)
1	5
2	8
3	11
4	14



b) Is the relation linear? Explain.

____

c) Should the points on the graph be joined with a line? Explain.

____

Example 2 Graphing a Linear Relation from an Equation

a) Complete the table of values.

b) Graph the relation represented by the data in the table of values.

c) Describe the patterns in the graph and in the table.

d) Is the relation linear? Explain.

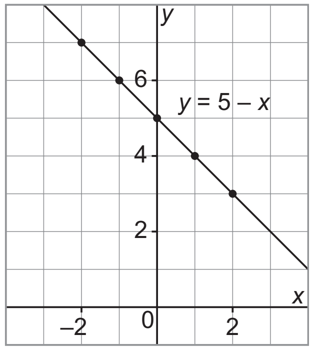
x	$y = 5 - x$
−2	
−1	
0	
1	
2	

Solution

a)

x	$y = 5 - x$
−2	$5 - (-2) = 7$
−1	$5 - (-1) = 6$
0	$5 - 0 = 5$
1	$5 - 1 = 4$
2	$5 - 2 = 3$

b)



Since we can substitute any value for x, the points can be joined with a line.

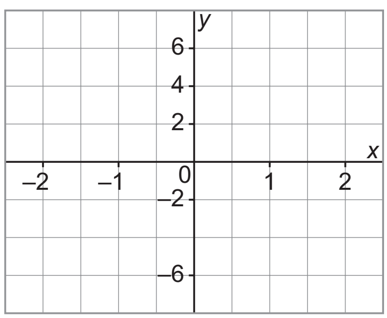
c) Each point on the graph is 1 unit right and 1 unit down from the previous point. In the table, when x increases by 1, y decreases by 1.

d) This is a linear relation because its graph is a straight line.

Check

1. Complete the table of values. Then, graph the relation.

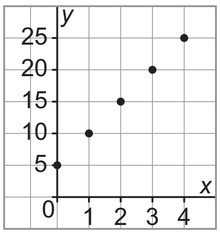
x	$y = 4x - 2$
−1	−6
0	−2
1	______
2	______



Practice

1. Which graphs represent a linear relation?

a)



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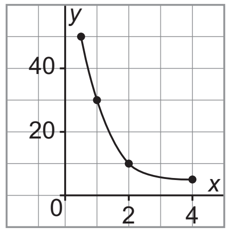
b)



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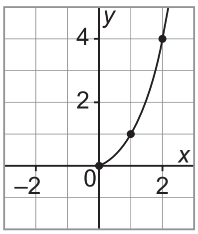
c)



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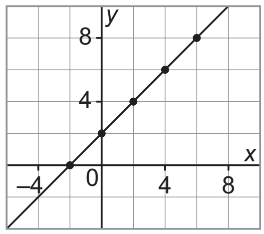
d)



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2. Describe the patterns in each table of values.
Does each table of values represent a linear relation?

a)

x	*y*
−3	6
−2	5
−1	4
0	3

x increases by _______ each time.
y decreases by _______ each time.
The relation is _______, because a constant change in *x* produces a constant change in *y*.

b)

x	*y*
0	1
2	4
4	7
6	10

x increases by _______ each time.
y increases by _______ each time.
The relation is _______, because a constant change in *x* produces a constant change in *y*.

c)

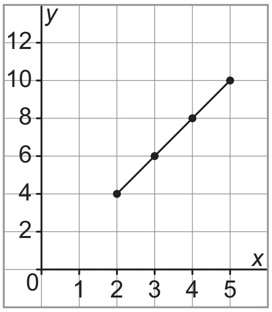
x	*y*
1	1
2	3
3	7
4	13

x increases by _______ each time.
y ____
The relation ____

3. Each graph and table of values represents a linear relation.

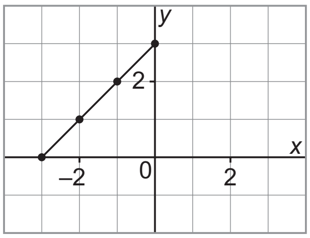
a) Complete each table of values.

i)



x	y
2	4
3	6
4	___
5	___

ii)



x	y
−3	0
−2	___
−1	2
0	___

b) Describe the patterns in the table.

i) When x increases by _______, y increases by _______.

ii) When _______ increases by _______, _______ increases by _______.

c) Describe the patterns in the graph.

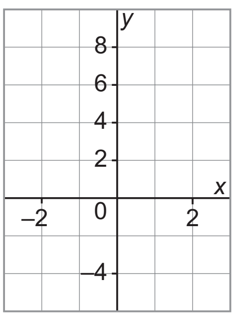
i) To get from one point to the next, move 1 unit right and __________ up.

ii) To get from one point to the next, move __________ right and __________ up.

4. Complete the table of values for each linear relation, then graph it.

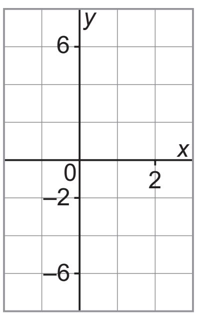
a) $y = 4x$

x	y
−1	___
0	___
1	___
2	___



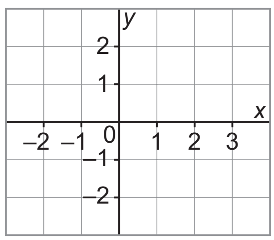
b) $y = -3x$

x	y
−1	___
0	___
___	___
___	___

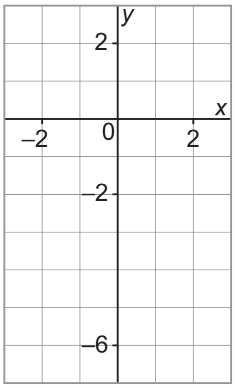


c) $y = 1 - x$

x	y
0	___
___	___
___	___
___	___



5. Complete the table of values.
Graph the data.



x	$y = 2x - 4$
−1	−6
0	−4
1	________
2	________

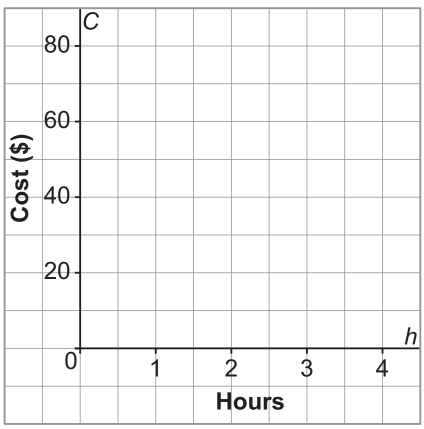
6. For special events, a bowling alley charges a set fee plus a fee for each hour bowled.

Bowling Costs

Hours, h	Cost, C ($)
1	40
2	50
3	60
4	70

a) Graph the data.

Bowling Costs



Does it make sense to join the points on the graph? Explain.

____

____

b) Is this a linear relation? Why?

____

____

c) Describe the pattern in words and using an equation.
When h increases by _______, C increases by _______.
C = _______h + 30

4.3 Skill Builder

Solving Equations

Solving an equation means finding a value of the variable that makes the equation true. To solve the equation $3x - 2 = 7$, find the value of x so that the left and the right side of the equation are balanced.

$3x - 2 = 7$	Add 2 to both sides to isolate x.
$3x - 2 + 2 = 7 + 2$	
$3x = 9$	Divide each side by 3.
$\frac{3x}{3} = \frac{9}{3}$	
$x = 3$	

To verify the equation, substitute $x = 3$ in the original equation.

Left side: $3(3) - 2$	Right side $= 7$
$= 9 - 2$	
$= 7$	

Since the left side equals the right side, the solution is correct.

Check

1. Solve each equation.

a) $2x + 3 = 11$

$2x + 3 -$ _____ $= 11 -$ _____

____

____

____

Check your solution.

b) $3 - 2x = -9$

____

____

____

____

4.3 Another Form of the Equation for a Linear Relation

FOCUS Recognize the equations of horizontal, vertical, and oblique lines, and graph them.

Example 1 Graphing and Describing Vertical Lines

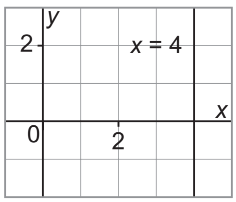
For the equation $x = 4$:

a) Draw the graph.
b) Describe the graph.

Solution

$x = 4$

a) For any value of y, x is always 4.



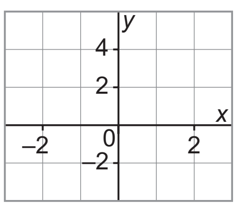
The vertical line intersects the x-axis at 4.

b) The graph is a vertical line.
Every point on the graph has x-coordinate 4.

When the equation is $x =$ a constant, the graph is a **vertical line.**

Check

1. For the equation $x = 1$:
Draw the graph.
Then describe the graph.



The graph is a ______________ line.
Every point on the graph has _____-coordinate _____.

Example 2 Graphing and Describing Horizontal Lines

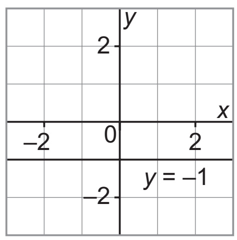
For the equation $y + 1 = 0$:

a) Draw the graph.
b) Describe the graph.

Solution

$y + 1 = 0$ Solve for y. Subtract 1 from each side.
$y + 1 - 1 = 0 - 1$
$y = -1$

a)



The horizontal line intersects the y-axis at –1.

b) The graph is a horizontal line.
Every point on the graph has y-coordinate -1.

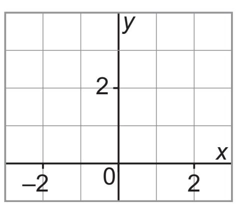
When the equation is y = a constant, the graph is a **horizontal line.**

Check

1. For the equation $y = 3$:

a) Draw the graph.
b) Describe the graph.

a)



b) The graph is a ________________.
Every point on the graph has _____-coordinate _____.

Example 3 Graphing an Equation

For the equation $y + 2x = 4$:

a) Make a table of values for $x = -2$, 0, and 2.

b) Graph the equation.

Solution

a) $y + 2x = 4$

Substitute each value of x, then solve for y.

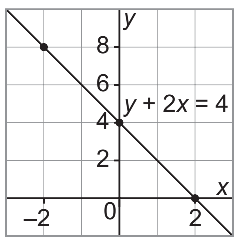
Substitute: $x = -2$
$y + 2(-2) = 4$
$y - 4 = 4$
$y - 4 + 4 = 4 + 4$
$y = 8$

Substitute: $x = 0$
$y + 2(0) = 4$
$y + 0 = 4$
$y = 4$

Substitute: $x = 2$
$y + 2(2) = 4$
$y + 4 = 4$
$y + 4 - 4 = 4 - 4$
$y = 0$

x	y
−2	8
0	4
2	0

b) Plot the points on a grid, and join the points.
The graph is an **oblique** line.



Oblique means slanted.

Check

1. Do not graph the equations.
Does each equation describe a horizontal line, a vertical line, or an oblique line?

a) $y = 4$ __________________

b) $y = 3x - 2$ __________________

c) $x = -1$ __________________

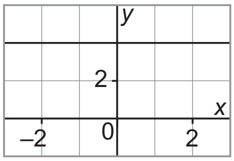
d) $2x + y = -6$ __________________

Practice

1. Which equation describes each graph?

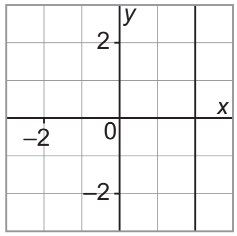
i) $y = 4$ **ii)** $x = 2$ **iii)** $y = 2$ **iv)** $x = 4$

a)



____

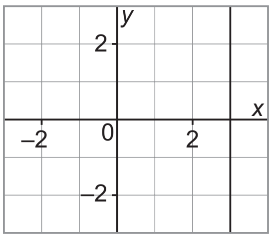
b)



____

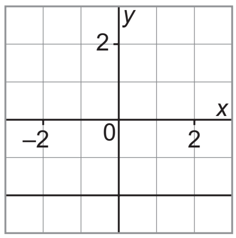
2. Write an equation to describe each line.

a)



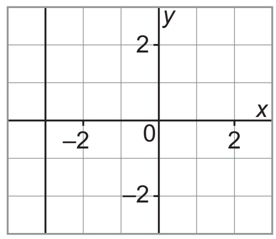
$x =$ ____________

b)



$y =$ ____________

c)



____

3. a) Is each line vertical or horizontal?

i) $x = -1$

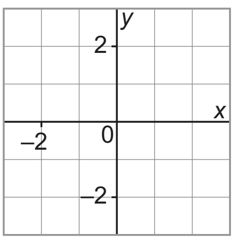
____

ii) $y = -4$

____

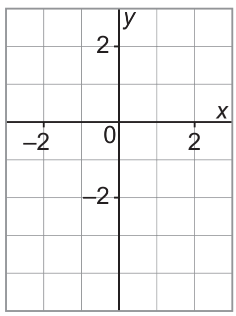
b) Graph each line. Describe the graph.

i)



Every point on the graph has x-coordinate ______.

ii)



Every point on the graph has ________-coordinate ________.

4. a) Does each equation describe a vertical line, a horizontal line, or an oblique line?

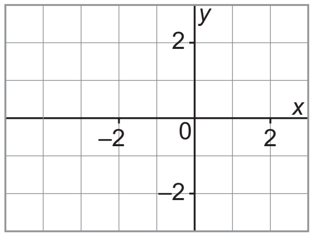
i) $x + 3 = -1$ ____________ **ii)** $1 + y = 0$ ____________ **iii)** $x + 2y = 8$ ____________

b) Graph the first 2 equations in part a.

i) $x + 3 = -1$

____

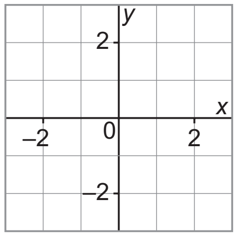
____



ii) $1 + y = 0$

____

____



c) For the equation $x + 2y = 8$:
Complete the table of values for $x = -2$, 0, and 2.
Graph the equation.

Substitute: $x = -2$
_____ $+ 2y = 8$

____

____

____

____

Substitute: $x = 0$
_____ $+ 2y = 8$

____

____

____

____

Substitute: $x = 2$
_____ $+ 2y = 8$

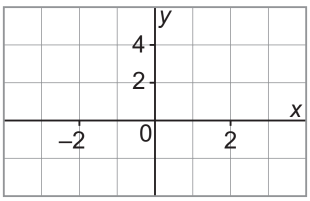
____

____

____

____

x	*y*
−2	__
0	__
2	__

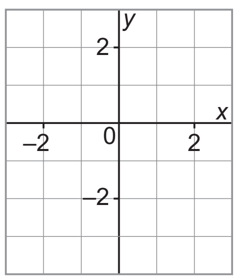


5. a) Explain why this equation describes the graph below.
$y + 3 = 0$

____

____

This is a ____________ line, with _____-coordinate _____, which matches the graph.



CHECKPOINT

Can you ...

- Use equations to describe and solve problems involving patterns?
- Graph a linear relation?
- Recognize the equations of horizontal, vertical, and oblique lines, and graph them?

4.1 **1.** This pattern of squares continues. Draw the next 2 figures in the pattern.

Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

a) Complete the table of values below.

	Figure Number, *n*	Number of Squares, *s*	
	1	1	
+1	2	5	+4
+1	3	____	+4
+1	4	____	+4
+1	5	____	+4

b) What patterns do you see?

The figure number increases by _____ each time.
The number of squares increases by _____ each time.

c) Describe how the number of squares relates to the figure number.

The number of squares is _____ times the figure number, less _____.

d) Write an equation for this pattern.
$s =$ _____n − _____

2. The pattern in the table of values continues. Complete the table.

Number of Red Buttons, *r*	Number of Blue Buttons, *b*
2	10
3	13
4	16
5	19
_____	_____
_____	_____

a) What patterns do you see? The number of red buttons increases by _____ each time. The number of blue buttons increases by _____ each time.

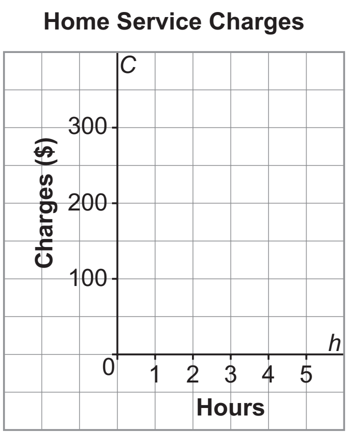
b) Write an equation that relates the number of blue buttons to the number of red buttons.
$b =$ _____$r +$ _____

4.2 **3.** A home service provider charges for the service according to the table of values.

Home Service Charges

Hours, *h*	Charges, *C* ($)
0	60
1	150
2	240
3	330

a) Graph the data.



b) Is this an example of a linear relation? Why?

____

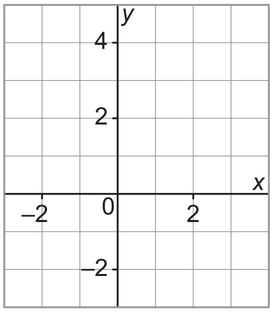
c) Describe the patterns in the table.
As *h* increases by _____, *C* increases by _____.

d) How is the pattern shown in the graph?
On the graph, to get from one point to the next, move 1 unit right, and ___________ up.

e) Write an equation for this pattern.
$C =$ _____$h +$ _____

4. a) This table of values represents a linear relation.
Graph the data.

x	*y*
−2	−2
−1	0
0	2
1	4



b) How do the patterns in the graph relate to the patterns in the table?
In the table, as *x* increases by _____, *y* increases by _____.
On the graph, to get from one point to the next,
move 1 unit right and _________ up.

c) Write an equation for this pattern.
$y =$ _____$x +$ _____

4.3 **5.** Does each equation describe a horizontal line, a vertical line, or an oblique line?

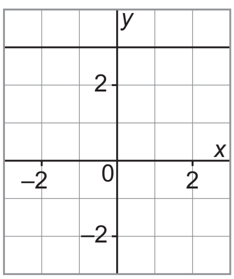
a) $x + 4 = 0$ ________________________

b) $y = -6$ ________________________

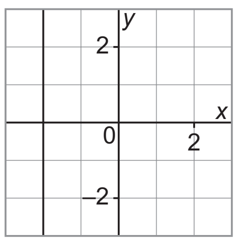
c) $x + y = 2$ ________________________

d) $2y = 4$ ________________________

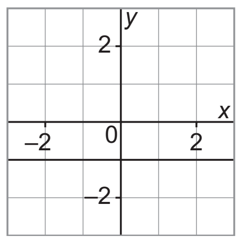
6. Write an equation to describe each line.

a)


____

b)


____

c)


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4.4 Matching Equations and Graphs

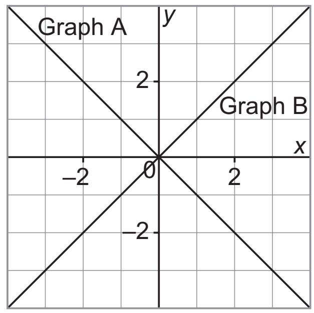
FOCUS Match equations and graphs of linear relations.

Example 1 Matching Equations with Graphs

Match each graph on the grid with its equation.

$y = x$

$y = -x$



Solution

Substitute $x = -1$, $x = 0$, and $x = 1$ in each equation.

$y = x$

x	*y*
−1	−1
0	0
1	1

We chose to use x-values of −1, 0, and 1 because they're often easy to substitute.

Points (−1, −1), (0, 0), and (1, 1) lie on Graph B.

So, $y = x$ matches Graph B.

$y = -x$

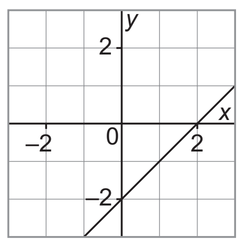
x	*y*
−1	1
0	0
1	−1

Points (−1, 1), (0, 0), and (1, −1) lie on Graph A.

So, $y = -x$ matches Graph A.

Check

1. Which equation describes the graph at the right?



$y = x + 2$

x	$y = x + 2$
0	$y = 0 + 2 =$ ____
1	$y =$ ____ $+ 2 =$ ____
2	$y =$ ____ $+ 2 =$ ____

$y = x - 2$

x	$y = x - 2$
0	$y =$ ____ $- 2 =$ ____
1	$y =$ ________ $=$ ____
2	$y =$ ________ $=$ ____

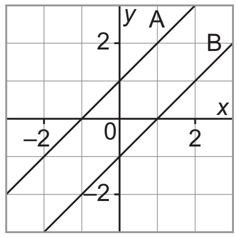
Points (______), (______), and (______) do not lie on the graph.

Points (______), (______), and (______) lie on the graph.

So, the equation $y =$ ______ describes the graph.

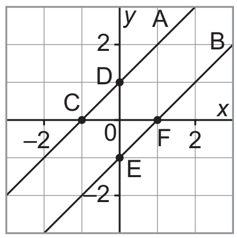
Example 2 Identifying a Graph Given Its Equation

Which graph on this grid has the equation $y = x - 1$?



Solution

Pick 2 points on each graph and check if their coordinates satisfy the equation.



Since C does not work, we do not have to check for D.

For Graph A, use: C(−1, 0) and D(0, 1)
Check if C(−1, 0) satisfies the equation $y = x - 1$.
Substitute $x = -1$ and $y = 0$ in $y = x - 1$
Left side: $y = 0$ Right side: $x - 1 = (-1) - 1$
$= -2$

The left side does not equal the right side.
So, Graph A does not have equation $y = x - 1$.

Verify that the other graph does match the equation.
For Graph B, use: E(0, −1) and F(1, 0)
Check if E(0, −1) satisfies the equation $y = x - 1$.
Substitute $x = 0$ and $y = -1$ in $y = x - 1$
Left side: $y = -1$ Right side: $x - 1 = 0 - 1$
$= -1$

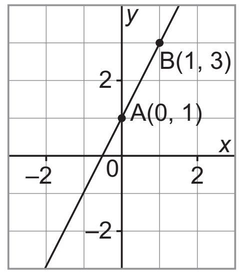
The left side does equal the right side.
So, E(0, −1) lies on the line represented by $y = x - 1$.

Check if F(1, 0) satisfies the equation $y = x - 1$.
Substitute $x = 1$ and $y = 0$ in $y = x - 1$
Left side: $y = 0$ Right side: $x - 1 = 1 - 1$
$= 0$

The left side does equal the right side.
So, F(1, 0) lies on the line represented by $y = x - 1$.
So, Graph B has equation $y = x - 1$.

Check

1. Show that this graph has equation $y = 2x + 1$.
   Use the points labelled on the graph.



   For A(0, 1): Substitute $x = 0$ and $y = 1$ in $y = 2x + 1$.
   Left side: $y =$ ___________ Right side: $2x + 1 =$ __________
   $=$ __________
   $=$ __________

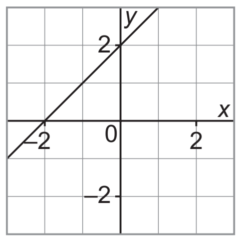
   For B(1, 3): Substitute $x =$ _______ and $y =$ _______ in $y = 2x + 1$.
   Left side: $y =$ _______ Right side: $2x + 1 =$ ___________
   $=$ ___________
   $=$ ___________

   ____

Practice

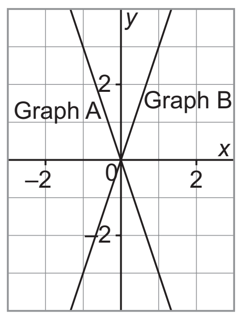
1. Show that the equation $y = x + 2$ matches the graph.
   Fill in the table of values.

x	$y = x + 2$
-2	$y = -2 + 2 =$ ____
-1	$y =$ ___ $+ 2 =$ ____
0	$y =$ ______ $=$ ____



   From the table:
   Points (_______), (_______), and (_______) lie on the graph.
   So, $y = x + 2$ matches the graph.

2. Match each equation with a graph.
   $y = 3x$ $y = -3x$



   Fill in the tables of values.

x	$y = 3x$
-1	$y = 3($___$) =$ ____
0	$y = 3($___$) =$ ____
1	$y = 3($___$) =$ ____

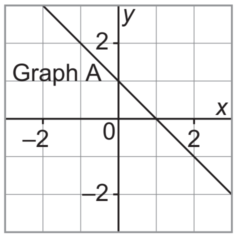
x	$y = -3x$
-1	$y = -3($___$) =$ ____
0	$y =$ ___(___) $=$ ____
1	$y =$ _______ $=$ ____

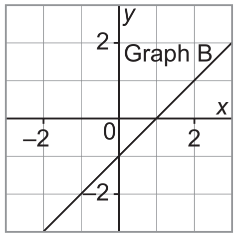
   From the tables:
   $y = 3x$ has points (_________), (_________), and (_________).
   These points lie on Graph _________.
   So, $y = 3x$ matches Graph _________.

   $y = -3x$ has points (_________), (_________), and (_________).
   These points lie on Graph _________.
   So, $y = -3x$ matches Graph _________.

3. Match each equation with a graph.

$y = 1 - x$ $\qquad$ $y = x - 1$





Fill in the tables of values.

x	$y = 1 - x$
-1	$y = 1 - (\_\_\_\_) = \_\_\_\_$
0	$y = 1 - \_\_\_\_ = \_\_\_\_$
1	$y = 1 - \_\_\_\_ = \_\_\_\_$

x	$y = x - 1$
-1	$y = \_\_\_\_\_\_\_\_ = \_\_\_\_$
0	$y = \_\_\_\_\_\_\_\_ = \_\_\_\_$
1	$y = \_\_\_\_\_\_\_\_ = \_\_\_\_$

From the tables:

$y = 1 - x$ has points (______), (______), and (______).

These points lie on Graph ______.

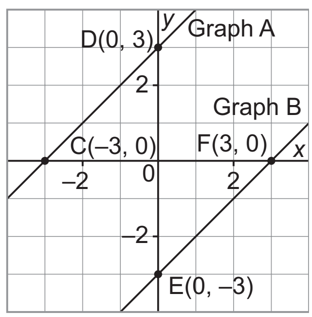
So, $y = 1 - x$ matches Graph ______.

$y = x - 1$ has points (______), (______), and (______).

These points lie on Graph ______.

So, $y = x - 1$ matches Graph ______.

4. Which graph has equation $y = x - 3$?



For C(-3, 0):

Left side: $y =$ _______ $\qquad$ Right side: $x - 3 =$ _______

$=$ _______

The left side __________ equal the right side.

____

For E(0, -3):

Left side: $y =$ _______ $\qquad$ Right side: $x - 3 =$ _______

$=$ _______

The left side ________ the right side.

For F(3, 0):

Left side: $y =$ _______ $\qquad$ Right side: $x - 3 =$ ________

$=$ ________

____

So, Graph _____ has equation $y = x - 3$.

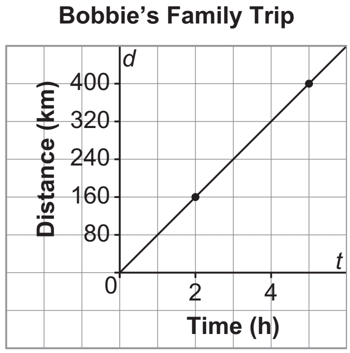
4.5 Using Graphs to Estimate Values

FOCUS **Use interpolation and extrapolation to estimate values on a graph.**

When we estimate values between 2 given data points on a graph of a linear relation, we use **interpolation.**

Example 1 Using Interpolation to Solve Problems

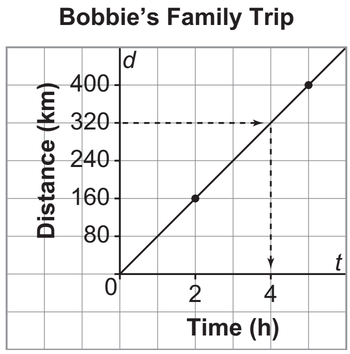
This graph shows the distance travelled by Bobbie's family on a trip from Calgary to Moose Jaw. How long did it take his family to travel 320 km?



Solution

To find how long it took to travel 320 km:
- Locate the point on the vertical axis that represents 320 km.
- Draw a horizontal line to the graph.
- Then draw a vertical line from the graph to the horizontal axis.

Read the value where the vertical line meets the horizontal axis.
It took about 4 h to travel 320 km.



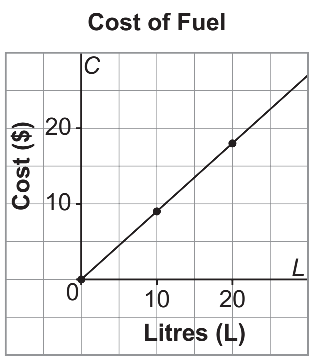
We could follow the same process to find that, after 3 h, the family has travelled about 240 km.

Check

1. Use the graph to find the following values.

a) The cost of 15 L of fuel.
About $______.

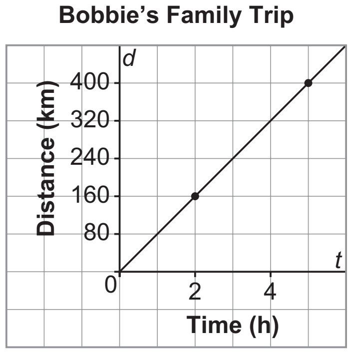
b) The quantity of fuel that can be purchased for $10.
About _______ L.



When we extend a graph of a linear relation to estimate values that lie beyond the graph, we use **extrapolation.**

Example 2 Using Extrapolation to Solve Problems

On his family trip from Calgary to Moose Jaw, Bobbie wants to predict how long it will take to travel 640 km.



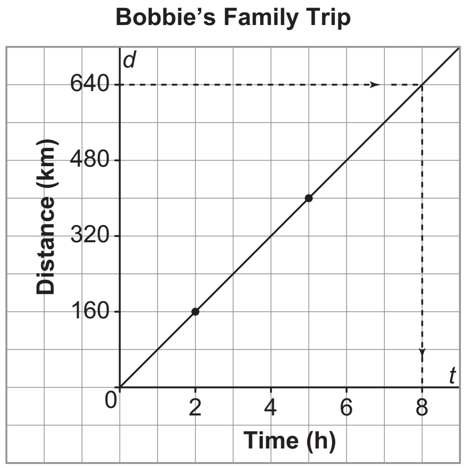
We assume that Bobbie's family will continue to travel at the same average speed.

Solution

Since the relation appears to be linear, we can extend the graph.

- Locate the point on the vertical axis that represents 640 km.
- Draw a horizontal line to the graph.
- Then draw a vertical line from the graph to the horizontal axis.

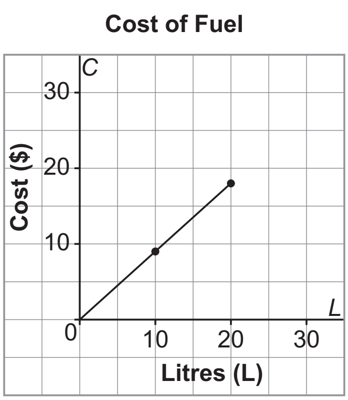
Read the value where the vertical line meets the horizontal axis. It will take about 8 h to travel 640 km.



Check

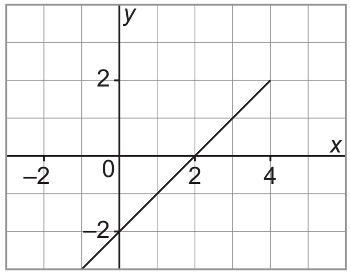
1. Use the graph to find the cost of 30 L of fuel.

____



Practice

1. Use this graph of a linear relation.



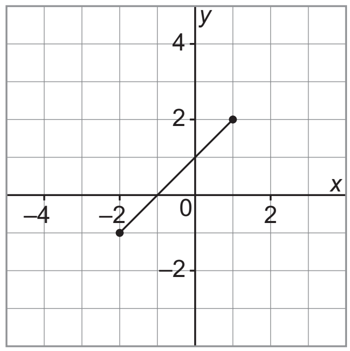
a) What is the value of x when $y = 3$?

$x =$ ________

b) What is the value of y when $x = 1$?

$y =$ ________

2. This graph shows a linear relation.



a) What is the value of x when $y = 4$?

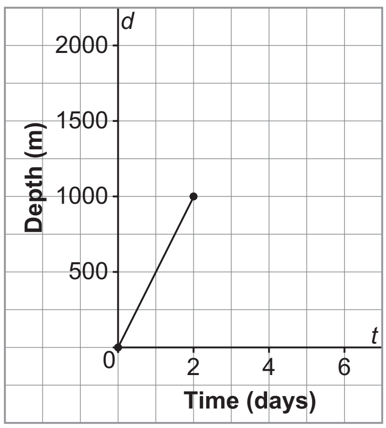
$x =$ ________

b) What is the value of y when $x = -4$?

$y =$ ________

3. This graph shows a linear relation for different drilling depths.

Drilling Depths



a) Estimate the depth drilled in 1 day.

About ________ m

b) Estimate the time taken to drill to a depth of 750 m.

About ________ days

c) Estimate the depth that will be drilled in 3 days.

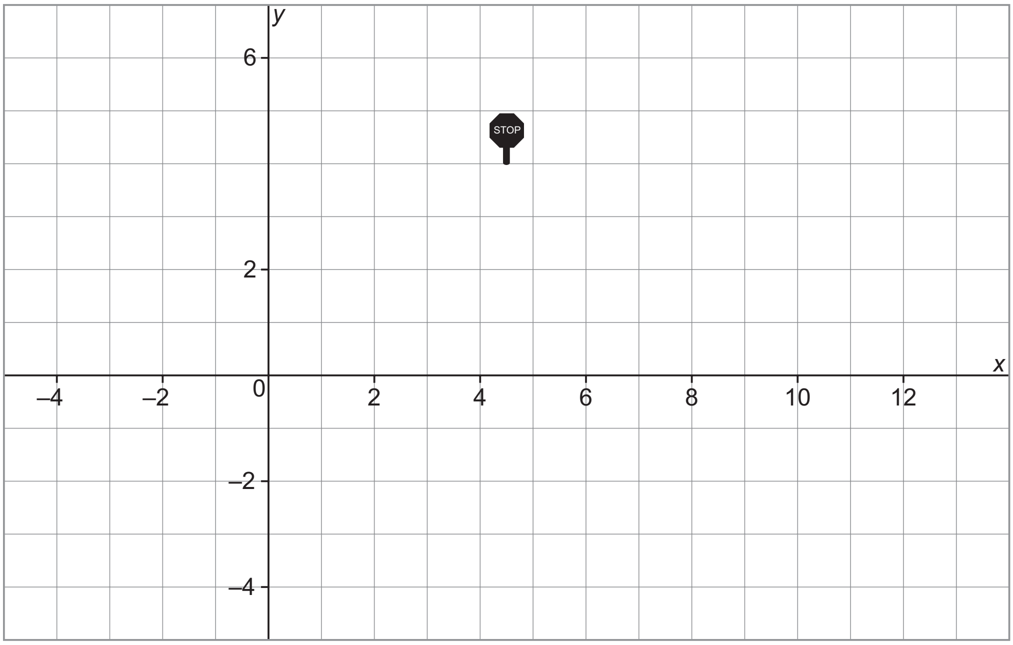
About ________ m

d) Estimate the time it will take to drill 2000 m.

About ________ days

Unit 4 Puzzle

A Graphing Perspective



1. On the grid, plot the lines represented by:

a) $y = 4$

b) $x = y$

x	*y*
−4	___
0	___
4	___

c)

x	*y*
5	4
9	0
13	−4

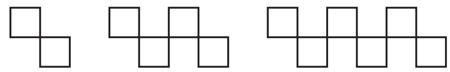
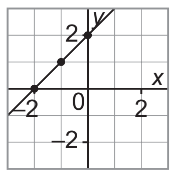
2. Plot (4.5, 4) and (4.5, 3), and join the points with a line.

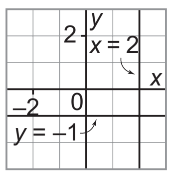
3. Plot (4.5, 2) and (4.5, 0), and join the points with a line.

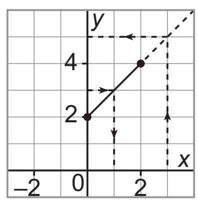
4. Plot (4.5, −1) and (4.5, −4), and join the points with a line.

What do you see?

Unit 4 Study Guide

<table>
<tr><th>Skill</th><th>Description</th><th>Example</th></tr>
<tr><td>Generalize a pattern</td><td>Recognize and extend a pattern using a drawing and a table of values.
Describe the pattern.
Write an equation for the pattern.</td><td>
Figure 1 Figure 2 Figure 3
<table><tr><th>Figure Number, n</th><th>Figure Value, v</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>6</td></tr></table>As the figure number increases by 1, the figure value increases by 2. The pattern is: multiply the figure number by 2 to get the figure value.
An equation is: $v = 2n$</td></tr>
<tr><td>Linear relations</td><td>The points on the graph of a linear relation lie on a straight line. To graph a linear relation, create a table of values first.
In a linear relation, a constant change in x produces a constant change in y.</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>−2</td><td>0</td></tr><tr><td>−1</td><td>1</td></tr><tr><td>0</td><td>2</td></tr></table>

As x increases by 1, y increases by 1.</td></tr>
<tr><td>Horizontal and vertical lines</td><td>A vertical line has equation $x = a$
A horizontal line has equation $y = b$</td><td>

The graph of $x = 2$ is a vertical line. Every point on the line has x-coordinate 2.
The graph of $y = -1$ is a horizontal line. Every point on the line has y-coordinate −1.</td></tr>
<tr><td>Interpolation and extrapolation</td><td>When we estimate values between 2 given points on a graph, we use interpolation.
When we estimate values beyond given points on a graph, we use extrapolation.</td><td>

When $y = 3$, $x = 1$
Extend the graph to find that, when $x = 3$, $y = 5$</td></tr>
</table>

Unit 4 Review

4.1 **1.** This pattern continues.

a) Draw the next 2 figures in the pattern.

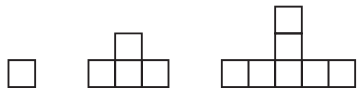


Figure 1 Figure 2 Figure 3 _____ Figure 4 _____ Figure 5

b) Complete the table of values.

Figure Number, *n*	Number of Squares, *s*
1	1
2	4
3	7
_____	_____
_____	_____

c) Describe the patterns in the table.
The figure number increases by _____ each time.
The number of squares increases by _____ each time.

d) Write an equation that relates the number of squares to the figure number.
$s = \_\_\_\_\_ n - \_\_\_\_\_$

e) What is the number of squares in figure 10?
When $n = 10$:
$s = \_\_\_\_\_\_ - 2 = \_\_\_\_\_\_ - 2 = \_\_\_\_\_\_$
There are ______ squares in figure 10.

2. The pattern in this table of values continues.

a) Complete the table.

b) Which expression below represents the number of squares in terms of the figure number? ________

i) $5n$

ii) $5n - 4$

iii) $n + 4$

iv) $n - 4$

Figure Number, *n*	Number of Squares, *s*
1	5
2	6
3	7
4	____
5	____

4.2 **3.** Complete each table of values.

a) $y = x + 1$

x	y
1	___
2	___
3	___
4	___

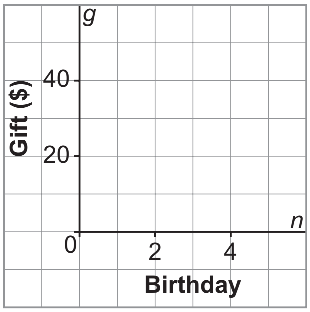
b) $y = x - 1$

x	y
2	___
4	___
6	___
8	___

4. On his first birthday, Hayden was given $20 by his grandfather. Each year's gift is $10 more than the year before. The data is given in the table below.

Grandfather's Gifts

Birthday, *n*	Gift, *g* ($)
1	20
2	30
3	40
4	50



a) Graph the data.

b) Is the graph linear? Explain your thinking.
The points ____________________, so the graph is ________.

c) Should the points be joined? Explain why or why not.

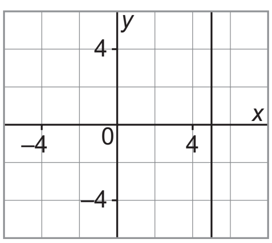
____

____

d) How are the patterns in the table shown in the graph? In the table, as the birthday increases by ____, the gift value increases by ____. Each point on the graph is ______________ and ______________ from the previous point.

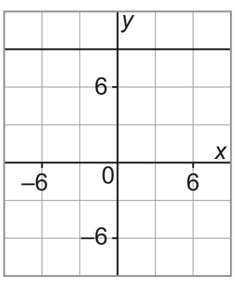
4.3 **5.** Write an equation to describe each line.

a)



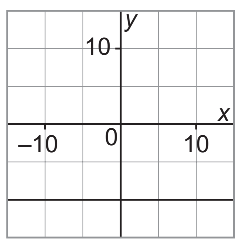
____

b)



____

c)



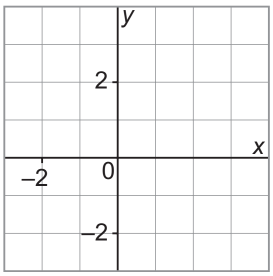
____

6. Does each equation represent a horizontal line, a vertical line, or an oblique line?

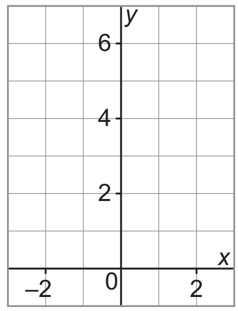
a) $x = 2$ ____________ **b)** $y = 2x + 2$ ____________ **c)** $y = 3$ ____________ **d)** $x = -1$ ____________

Draw a graph for each equation above.

a)

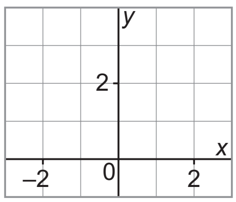


b)

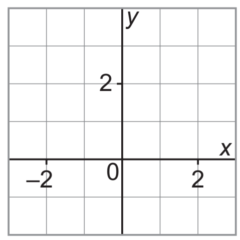


x	***y***
____	____
____	____
____	____

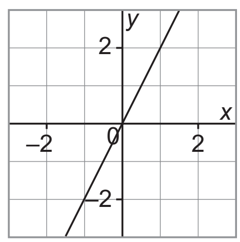
c)



d)



4.4 **7.** Which equation describes the graph?
$y = 2x$ or $y = -2x$
Fill in the tables of values.

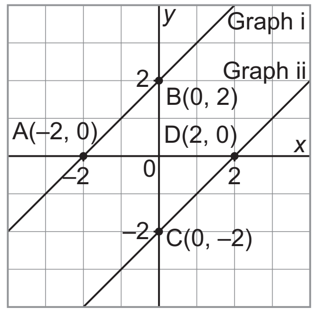


x	***y* = 2*x***
−1	2(____) = ____
0	2(____) = ____
1	2(____) = ____

x	***y* = −2*x***
−1	−2(____) = ____
0	________ = ____
1	________ = ____

From the tables:
$y = 2x$ has points (________), (________), and (________).
$y = -2x$ has points (________), (________), and (________).
The graph passes through the points (________), (0, 0), and (________).
So, $y =$ _______ describes the graph.

8. Which graph represents the equation $x - y = 2$?



For A(−2, 0):

Left side: $x - y =$ __________ Right side: __________

$=$ __________

The left side ____________ equal the right side.

____

For C(0, −2):

Left side: $x - y =$ ____________ Right side: __________

$=$ ____________

The left side __________ the right side.

For D(2, 0):

____

____

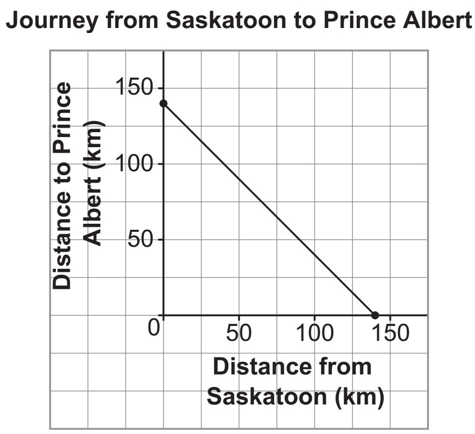
____

So, Graph ________ has equation $x - y = 2$.

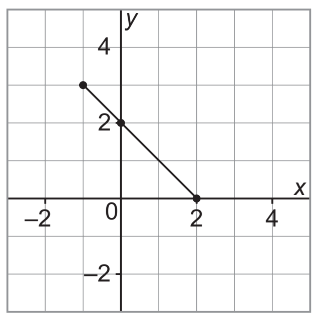
4.5 **9.** This graph shows Emma's and Julianna's journey from Saskatoon to Prince Albert.

When Emma and Julianna have travelled 100 km, about how far do they still have to go?

____



10. This graph represents a linear relation.



a) Estimate the value of y when:

i) $x = 0$ ____________ **ii)** $x = 1$ ____________

b) Estimate the value of x when:

i) $y = 4$ ____________ **ii)** $y = -2$ ____________