

UNIT  
**5**

# Polynomials

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## What You'll Learn

- Recognize, write, describe, and classify polynomials.
- Represent polynomials using tiles, pictures, and algebraic expressions.
- Add and subtract polynomials.
- Multiply and divide a polynomial by a monomial.

## Why It's Important

Polynomials are used by

- homeowners to calculate mortgage and car payments
- computer technicians to encode information, such as PIN numbers for ATM machines and debit cards



## Key Words

term	monomial
variable term	binomial
constant term	trinomial
variable	simplify a polynomial
coefficient of the variable	like terms
polynomial	unlike terms
degree of a polynomial	distributive property

## 5.1 Skill Builder

### Modelling Expressions

We can use algebra tiles to model an expression.

One  represents  $+1$ . One  represents  $-1$ .

One  represents any variable, such as  $x$  or  $n$ .

One  represents  $-x$  or  $-n$ .


There are 2 .

They represent  $2x$ .

So, the tiles represent the expression  $2x - 1$ .



There is 1 .

It represents  $-1$ .

There are 3 .


They represent  $-3a$ .

So, the tiles represent the expression  $-3a + 2$ .



There are 2 .

They represent  $+2$ .



*We can use any letter as the variable.*

### Check

1. Which expression does each set of tiles represent?

a)   \_\_\_\_\_

b)   \_\_\_\_\_

c)   \_\_\_\_\_

d)   \_\_\_\_\_

2. Sketch algebra tiles to model each expression.

a)  $s + 4$

b)  $5b - 3$


c)  $-4n + 5$

d)  $-6w - 1$

# 5.1 Modelling Polynomials

**FOCUS** Model, write, and classify polynomials.

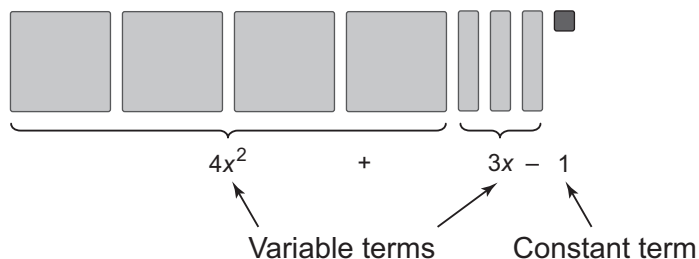
Some expressions contain  $x^2$  terms.

We use  to represent  $x^2$ .

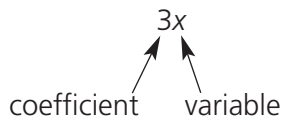
*When the variable is  $n$ , the tile is called the  $n^2$ -tile.*

We use  to represent  $-x^2$ .

For the expression  $4x^2 + 3x - 1$ :



In the term  $3x$ , the **variable** is  $x$  and the **coefficient of the variable** is 3.



An algebraic expression like this one is also called a **polynomial**.

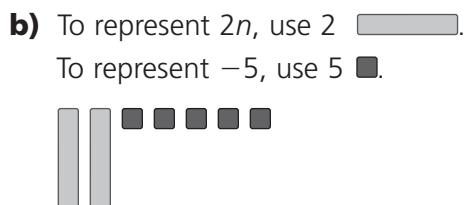
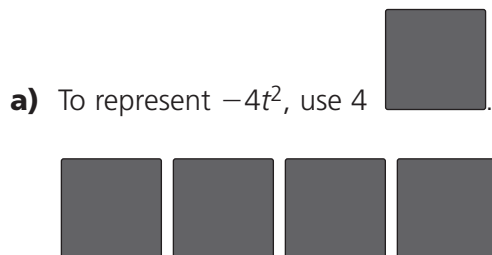
## Example 1 Modelling Polynomials with Algebra Tiles

Use algebra tiles to model each polynomial.

a)  $-4t^2$

b)  $2n - 5$

### Solution



## Check

1. Sketch algebra tiles to model each polynomial.

a)  $-3$

b)  $2x + 3$

c)  $2e^2 - e + 2$

d)  $-3d^2 + 2d - 5$

### Example 2 Recognizing the Same Polynomials in Different Variables

Which of these polynomials can be represented by the same algebra tiles?

a)  $2x^2 + 7x - 4$

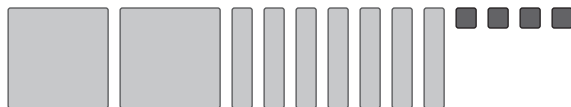
b)  $-4 + 2b^2 - 7b$

c)  $7s - 4 + 2s^2$

#### Solution

Select the tiles that match each term.

a)  $2x^2 + 7x - 4$



b)  $-4 + 2b^2 - 7b$



c)  $7s - 4 + 2s^2$



The variable used to name a tile does not matter.

In parts a and c, the same algebra tiles are used.

Since  $2x^2 + 7x - 4$  and  $7s - 4 + 2s^2$  can be represented

by the same tiles, the expressions represent the same polynomial.

*The order in which the terms are written does not matter.*

## Check

1. Which of these polynomials can be represented by the same algebra tiles?

a)  $3s^2 - 2s + 5$

b)  $5 - 3a^2 - 2a$

c)  $-2c + 5 - 3c^2$

The same tiles are used in parts \_\_\_\_\_ and \_\_\_\_\_.

So, \_\_\_\_\_ and \_\_\_\_\_ represent the same polynomial.

There are different **types** of polynomials, depending on the number of terms.

The **degree of a polynomial** tells you the greatest exponent of any term.

Type	Number of Terms	Example	Model	Degree
Monomial	1	$2s^2$		2
		$-2n$		1
		4		0
Binomial	2	$x^2 + 3$		2
		$2a - 1$		1
		$-2b^2 + 3b$		2
Trinomial	3	$-c^2 + 4c - 2$		2

A monomial has 1 type of tile.

*A constant term has degree 0.*

A binomial has 2 different types of tiles.

A trinomial has 3 different types of tiles.

An algebraic expression that contains a term with a variable in the denominator, such as  $\frac{5}{n}$ , or the square root of a variable, such as  $\sqrt{n}$ , is not a polynomial.

## Practice

1. Sketch algebra tiles to model each polynomial.

a)  $a^2 + 6$

b)  $y^2 - y + 3$

\_\_\_\_\_

\_\_\_\_\_

c)  $-2m^2 + 3m - 4$

d)  $2x^2 + 5x + 4$

\_\_\_\_\_

\_\_\_\_\_

2. Is the polynomial a monomial, binomial, or trinomial?

a)  $-7t$       The polynomial has \_\_\_ term, so it is a \_\_\_\_\_.

b)  $8d^2 + 7$       The polynomial has \_\_\_ terms, so it is a \_\_\_\_\_.

c)  $s^2 + 5s - 6$       The polynomial has \_\_\_ terms, so it is a \_\_\_\_\_.

d)  $4t - 12$       The polynomial has \_\_\_ terms, so it is a \_\_\_\_\_.

e)  $-15$       The polynomial has \_\_\_ term, so it is a \_\_\_\_\_.

3. Name the degree of each polynomial.

a)  $5a^2 - 3a + 6$       The term with the greatest exponent is  $5a^2$ .  
It has exponent \_\_\_\_\_.  
So, the polynomial has degree \_\_\_\_\_.

b)  $4b - 6$       The term with the greatest exponent is \_\_\_\_\_.  
It has exponent \_\_\_\_\_.  
So, the polynomial has degree \_\_\_\_\_.

c)  $4d^2 - 3d$       The term with the greatest exponent is \_\_\_\_\_.  
It has exponent \_\_\_\_\_.  
So, the polynomial has degree \_\_\_\_\_.

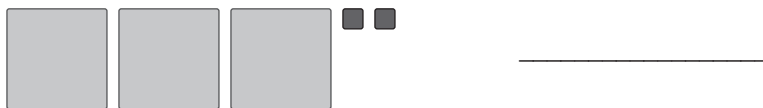
d)  $-4$        $-4$  can be written as  $-4x$ \_\_\_\_\_.  
So, the polynomial has degree \_\_\_\_\_.

4. Write the polynomial represented by each set of tiles.

a) Use the variable  $f$ .



b) Use the variable  $n$ .



c) Use the variable  $p$ .



5. Choose a set of tiles from question 4.

Write another polynomial that can be represented by the same set of tiles.

\_\_\_\_\_

\_\_\_\_\_

6. Identify the polynomials that can be represented by the same set of algebra tiles.

a)  $x^2 + 3x - 1$



b)  $4r^2 - 5r + 9$

\_\_\_\_\_

c)  $9 + 4z^2 - 5z$

\_\_\_\_\_

d)  $3s + 1 + s^2$

\_\_\_\_\_

Parts \_\_\_\_ and \_\_\_\_ use the same algebra tiles.

So, \_\_\_\_\_ and \_\_\_\_\_ both represent the same polynomial.



## 5.2 Skill Builder

### Modelling Integers

One  represents +1.

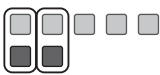
One  represents -1.



One  and one  combine to model 0.

 } +1  
 } -1      We call this a **zero pair**.

We can model any integer in many ways.

Each set of tiles below models +3.



Each pair of 1  and 1  makes a zero pair.

### Check

1. Write the integer modelled by each set of tiles.



\_\_\_\_\_



\_\_\_\_\_



\_\_\_\_\_



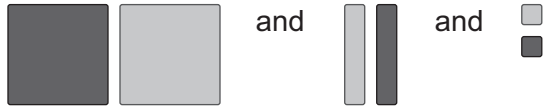
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## 5.2 Like Terms and Unlike Terms

**FOCUS** Simplify polynomials by combining like terms.

These are all zero pairs:

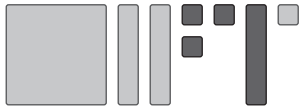


We can use zero pairs to simplify algebraic expressions.

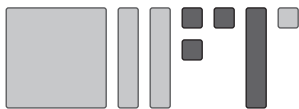
### Example 1 Combining Like Tiles and Removing Zero Pairs

Simplify this tile model.

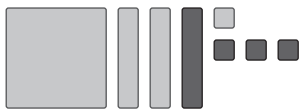
Write the polynomial that the remaining tiles represent.



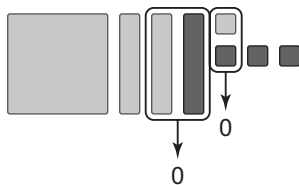
#### Solution



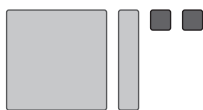
Group like tiles.



Remove zero pairs.



The tiles that remain are:



They represent:  $x^2 + x - 2$

*Matching tiles have the same size and shape.*

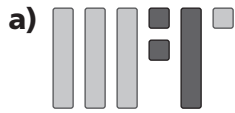
*When there is only 1 of a type of tile, we omit the coefficient 1.*

## Check

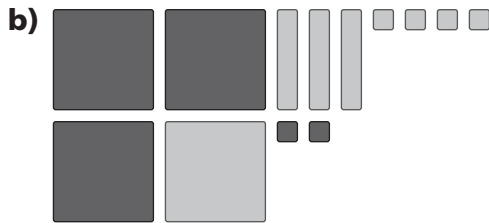
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1. Simplify each tile model.

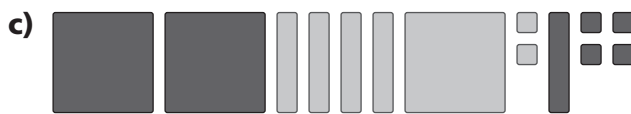
Write the polynomial that the remaining tiles represent.



Remaining tiles: \_\_\_\_\_ Polynomial: \_\_\_\_\_



Remaining tiles: \_\_\_\_\_ Polynomial: \_\_\_\_\_



Remaining tiles: \_\_\_\_\_ Polynomial: \_\_\_\_\_

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Terms that can be represented by matching tiles are called **like terms**.

Like terms:  $x^2$  and  $-2x^2$        $4s$  and  $-s$        $6$  and  $-2$        $5w$  and  $w$

Unlike terms:  $3s$  and  $s^2$        $2x$  and  $-5$        $3d^2$  and  $7$

We can **simplify a polynomial** by adding the coefficients of like terms.

To simplify  $-5x + 2x$ , add the integers:  $-5 + 2 = -3$

So,  $-5x + 2x = -3x$

## Example 2 Simplifying a Polynomial Symbolically

Simplify:

a)  $3a + 6 + a - 4$

b)  $-x^2 + 4x - 5 + 3x^2 - 4x + 1$

### Solution

a)  $3a + 6 + a - 4$   
 $= 3a + 1a + 6 - 4$   
 $= 4a + 2$

Group like terms.  
Add the coefficients of like terms.

b)  $-x^2 + 4x - 5 + 3x^2 - 4x + 1$   
 $= -x^2 + 3x^2 + 4x - 4x - 5 + 1$   
 $= 2x^2 + 0x - 4$   
 $= 2x^2 - 4$

Group like terms.  
Add the coefficients of like terms.

*We omit a term when its coefficient is 0.*

### Check

1. Simplify each polynomial.

a)  $5d + 2 + 3d - 1$   
 $= 5d + 3d + 2 - 1$   
 $= \underline{\hspace{1cm}}d + \underline{\hspace{1cm}}$

Group like terms.  
Add the coefficients of like terms:  
 $5 + 3 = \underline{\hspace{1cm}}$  and  $2 + (-1) = \underline{\hspace{1cm}}$

b)  $2a^2 - 3a + 5a^2 + 7a$   
 $= \underline{\hspace{3cm}}$   
 $= \underline{\hspace{3cm}}$

Group like terms.  
Add the coefficients of like terms:  
 $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$  and  $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

c)  $-x^2 + 4x - 5 + 2x^2 + x + 3$   
 $= \underline{\hspace{3cm}}$   
 $\underline{\hspace{3cm}}$   
 $\underline{\hspace{3cm}}$

*We omit the coefficient when it is 1.*

d)  $2x^2 + 6x + 7 - 2x^2 + 7x - 11$   
 $= \underline{\hspace{3cm}}$   
 $\underline{\hspace{3cm}}$   
 $\underline{\hspace{3cm}}$

## Practice

1. What is the coefficient of each term?

- a)**  $2x^2$  \_\_\_\_\_      **b)**  $6w$  \_\_\_\_\_      **c)**  $-3x$  \_\_\_\_\_  
**d)**  $7t$  \_\_\_\_\_      **e)**  $b$  \_\_\_\_\_      **f)**  $-s$  \_\_\_\_\_

2. a) Which of these terms are like  $3z^2$ ?

$5z$      $-z^2$      $-9$      $-6z$      $2z^2$      $-11$      $-4z^2$

$3z^2$  has variable \_\_\_\_\_ and exponent \_\_\_\_\_.

Find all terms with the same variable and exponent: \_\_\_\_\_

b) Which of these terms are like  $-5x$ ?

$-4x$      $-3x^2$      $-2$      $7x$      $5x^2$      $8$      $-x$      $-5t$

$-5x$  has variable \_\_\_\_\_ and exponent \_\_\_\_\_.

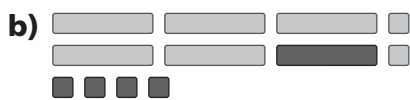
Find all terms with the same variable and exponent: \_\_\_\_\_

3. Simplify each tile model.

Write the polynomial that the remaining tiles represent.



Remaining tiles: \_\_\_\_\_      Polynomial: \_\_\_\_\_



Remaining tiles: \_\_\_\_\_      Polynomial: \_\_\_\_\_



Remaining tiles: \_\_\_\_\_      Polynomial: \_\_\_\_\_

4. Add integers to combine like terms.

a)  $-3c + 5c$        $-3 + 5 = \underline{\hspace{2cm}}$   
 $\hspace{1.5cm}$   $-3c + 5c = \underline{\hspace{2cm}}$

b)  $4s - s$        $4 + (-1) = \underline{\hspace{2cm}}$   
 $\hspace{1.5cm}$   $4s - s = \underline{\hspace{2cm}}$

c)  $-2x^2 + 7x^2$        $\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$   
 $\hspace{1.5cm}$   $\underline{\hspace{2cm}}$

d)  $8e^2 - 8e^2$        $\underline{\hspace{2cm}}$   
 $\hspace{1.5cm}$   $\underline{\hspace{2cm}}$

5. Simplify each polynomial.

a)  $5m + 7 - 2m + 1$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

Group like terms.  
Add the coefficients of like terms.

b)  $7c^2 - 6c - 4c^2 + c$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

Group like terms.  
Add the coefficients of like terms.

c)  $11 - 9v + v^2 + 2 - v$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   
 $\underline{\hspace{2cm}}$

*We usually write a polynomial so the exponents of the variable decrease from left to right.*

d)  $-7f^2 + 12f - 2 - 3f^2 - 3f + 5$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

*A polynomial in simplified form is equal to the original polynomial.*

6. Identify and explain any errors you find.

a)  $3x + 2 = 5x$        $\underline{\hspace{2cm}}$   
 $\hspace{1.5cm}$   $\underline{\hspace{2cm}}$

b)  $5s + 3s = 8s^2$        $\underline{\hspace{2cm}}$   
 $\hspace{1.5cm}$   $\underline{\hspace{2cm}}$   
 $\hspace{1.5cm}$   $\underline{\hspace{2cm}}$

c)  $x^2 - x^2 = 0$        $\underline{\hspace{2cm}}$   
 $\hspace{1.5cm}$   $\underline{\hspace{2cm}}$

## 5.3 Skill Builder

### Adding Integers

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To add two integers:  $3 + (-4)$

We can model each integer with tiles.

Circle zero pairs.

3:   
-4: 

There are 3 zero pairs.

There is 1 tile left.

It models  $-1$ .

So,  $3 + (-4) = -1$

*This is an addition sentence.*

### Check

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1. Sketch tiles to show the sum of each pair of integers.

Write an addition sentence each time.

a) 4:

5:

---

b) 6:

-2:

---

c) -3:

-5:

---

d) 3:

-3:

---

e) 5:

-8:

---

## 5.3 Adding Polynomials

**FOCUS** Use different strategies to add polynomials.

### Example 1 Adding Polynomials with Algebra Tiles

Use algebra tiles to model  $(3s^2 + 2s - 6) + (-s^2 - 2s + 1)$ .

Write an addition sentence.

#### Solution

Model each polynomial.

$$3s^2 + 2s - 6$$



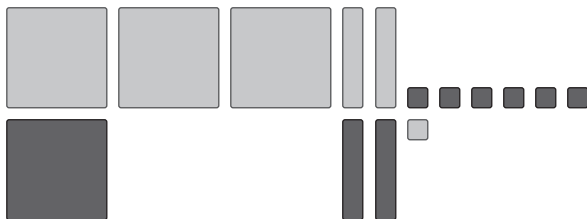
$$-s^2 - 2s + 1$$



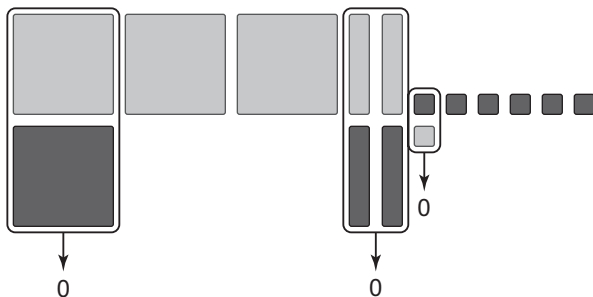
Combine the tiles.



Group matching tiles.



Remove zero pairs.



The remaining tiles are:



They represent:  $2s^2 - 5$

The addition sentence is:  $(3s^2 + 2s - 6) + (-s^2 - 2s + 1) = 2s^2 - 5$

## Check

1. Sketch algebra tiles to model each sum.

Then write the sum.

**a)**  $(6p + 4) + (-2p + 1)$

Remaining tiles: \_\_\_\_\_

So,  $(6p + 4) + (-2p + 1) =$  \_\_\_\_\_

**b)**  $(2x^2 - x + 1) + (x^2 - 3)$

Remaining tiles: \_\_\_\_\_

So,  $(2x^2 - x + 1) + (x^2 - 3) =$  \_\_\_\_\_

**c)**  $(3e^2 + 6e - 5) + (-4e^2 - 3e + 8)$

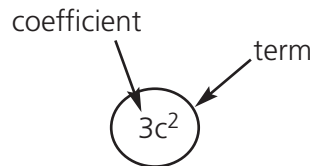
Remaining tiles: \_\_\_\_\_

So,  $(3e^2 + 6e - 5) + (-4e^2 - 3e + 8) =$  \_\_\_\_\_

Algebra tiles are not always available.

To add polynomials without tiles:

- remove the brackets
- add the coefficients of like terms



### Example 2 Adding Polynomials Symbolically

Add:  $(3c^2 + 5c - 6) + (2c^2 - 3c + 4)$

#### Solution

$$(3c^2 + 5c - 6) + (2c^2 - 3c + 4)$$

Remove the brackets.

$$= 3c^2 + 5c - 6 + 2c^2 - 3c + 4$$

Group like terms.

$$= \underline{3c^2 + 2c^2} + \underline{5c - 3c} - \underline{6 + 4}$$

Add the coefficients of like terms.

$$= 5c^2 + 2c - 2$$

*$3c^2$  and  $2c^2$  are like terms.*



## Check

1. Add.

$$\begin{aligned}\text{a) } (7g - 8) + (3g + 1) \\ &= 7g - 8 + 3g + 1 \\ &= \underline{7g + 3g - 8 + 1} \\ &= \underline{\hspace{2cm}}\end{aligned}$$

$$\begin{aligned}\text{b) } (2a^2 - 9a) + (-5a^2 + 12a) \\ &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}}\end{aligned}$$

$$\begin{aligned}\text{c) } (-c^2 + 11c - 3) + (4c^2 + 5) \\ &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}}\end{aligned}$$

Remove the brackets.

Group like terms.

Add the coefficients of like terms.

$$7 + 3 = \underline{\hspace{1cm}} \text{ and } -8 + 1 = \underline{\hspace{1cm}}$$

Remove the brackets.

Group like terms.

Add the coefficients of like terms.

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ and } \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

*Recall:  $-c^2$  has coefficient  $-1$ .*

We can also add 2 polynomials by aligning like terms vertically.

### Example 3 Adding Polynomials Vertically

Add:  $(2m + 9) + (3m^2 + m - 14)$

#### Solution

To add the polynomials, remove the brackets and align like terms vertically.

In  $3m^2 + m - 14$ , the term  $m$  has coefficient 1, so write  $m$  as  $1m$ .

$$\begin{array}{r} 2m + 9 \\ + 3m^2 + 1m - 14 \\ \hline 3m^2 + 3m - 5 \end{array}$$

Add the coefficients of like terms.

$$\begin{array}{r} 0 \quad 2 \quad 9 \\ +3 \quad +1 \quad +(-14) \\ \hline 3 \quad 3 \quad -5 \end{array}$$

So,  $(2m + 9) + (3m^2 + m - 14) = 3m^2 + 3m - 5$

## Check

1. Add vertically.

**a)**  $(2x + 3) + (4x + 8)$

$$\begin{array}{r} 2x + 3 \\ + 4x + 8 \\ \hline \end{array}$$

\_\_\_\_\_x + \_\_\_\_\_

**b)**  $(5p^2 + 12) + (-2p^2 + 3p - 7)$

$$\begin{array}{r} 5p^2 \qquad \qquad + 12 \\ + -2p^2 + 3p - 7 \\ \hline \end{array}$$

**c)**  $(-6b^2 - 2b + 8) + (9b - b^2 - 19)$

\_\_\_\_\_

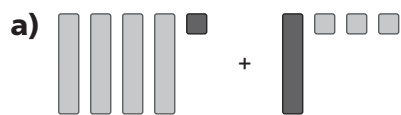
\_\_\_\_\_

\_\_\_\_\_

## Practice

1. Write the addition sentence modelled by each set of tiles.

Use the variable  $x$ .



2. Sketch algebra tiles to model each sum.

Then write the sum.

**a)**  $(-5w + 8) + (7w - 3) =$  \_\_\_\_\_

Remaining tiles: \_\_\_\_\_

**b)**  $(-6t^2 - 3t + 2) + (4t^2 - t + 1) =$  \_\_\_\_\_

Remaining tiles: \_\_\_\_\_

3. Add horizontally.

a)  $(2r - 3) + (3r - 1)$   
 $= 2r - 3 + 3r - 1$   
 $= 2r + 3r - 3 - 1$   
 $= \underline{\quad}r - \underline{\quad}$

b)  $(7h^2 - 2h) + (-4h^2 + 9h - 4)$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

Remove the brackets.

Group like terms.

Add the coefficients of like terms.

$2 + 3 = \underline{\quad}$  and  $-3 + (-1) = \underline{\quad}$

c)  $(-2y^2 + 6y - 1) + (2y^2 - 6y + 5)$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

4. Add vertically.

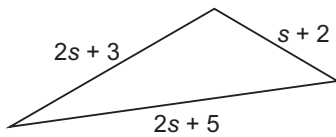
a)  $(9r + 7) + (2r - 3)$   
 $\begin{array}{r} 9r + 7 \\ + 2r - 3 \\ \hline \end{array}$   
 $\underline{\quad}r + \underline{\quad}$

b)  $(-a^2 + 4a) + (-3a^2 + 2a - 5)$   
 $\begin{array}{r} -a^2 + 4a \\ + -3a^2 + 2a - 5 \\ \hline \end{array}$   
 $\underline{\hspace{2cm}}$

c)  $(8v - 2v^2 - 3) + (9 + 6v^2 - 10v)$

$\underline{\hspace{2cm}}$   
 $\underline{\hspace{2cm}}$   
 $\underline{\hspace{2cm}}$

5. Find the perimeter of this triangle.



Perimeter =  $\underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

**Perimeter** is the distance around a shape. To find the perimeter, add the side lengths.

Remove the brackets.

Group like terms.

Add coefficients of like terms.

## 5.4 Skill Builder

### Subtracting Integers Symbolically

To subtract an integer without tiles, we add the opposite integer.  
 $-3$  and  $3$ ,  $-6$  and  $6$ , and  $-15$  and  $15$  are opposite integers.

To subtract:  $(-4) - (-3)$

Add the opposite integer.

The opposite of  $-3$  is  $3$ .

And,  $(-4) + 3 = -1$

So,  $(-4) - (-3) = -1$

We can use algebra tiles to check:

Model  $-4$ :



Take away  $-3$ :



One dark gray square remains.

So,  $(-4) - (-3) = -1$

*We omit the + sign when the integer is positive.*

### Check

#### 1. Subtract.

**a)**  $6 - (-2)$ :

The opposite of  $-2$  is \_\_\_\_.

Add the opposite:  $6 + \underline{\quad} = \underline{\quad}$

So,  $6 - (-2) = \underline{\quad}$

**b)**  $3 - (4)$ :

The opposite of  $4$  is \_\_\_\_.

Add the opposite: \_\_\_\_\_

So,  $3 - (4) = \underline{\quad}$

**c)**  $(-8) - (-5)$ :

The opposite of  $-5$  is \_\_\_\_.

Add the opposite: \_\_\_\_\_

So, \_\_\_\_\_

**d)**  $(-9) - (4)$ :

The opposite of  $4$  is \_\_\_\_.

Add the opposite: \_\_\_\_\_

So, \_\_\_\_\_

## 5.4 Subtracting Polynomials

**FOCUS** Use different strategies to subtract polynomials.

To subtract a polynomial, we subtract each term of the polynomial.

### Example 1 Subtracting Polynomials with Algebra Tiles

Use algebra tiles to model  $(3b^2 - 2b - 1) - (-2b^2 - b + 2)$ .

Write a subtraction sentence.

#### Solution

Model:  $3b^2 - 2b - 1$



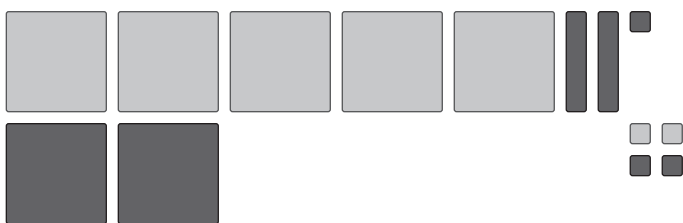
To subtract  $-2b^2 - b + 2$ , take away 2 , 1 , and 2 .

There are no  or  to take away.

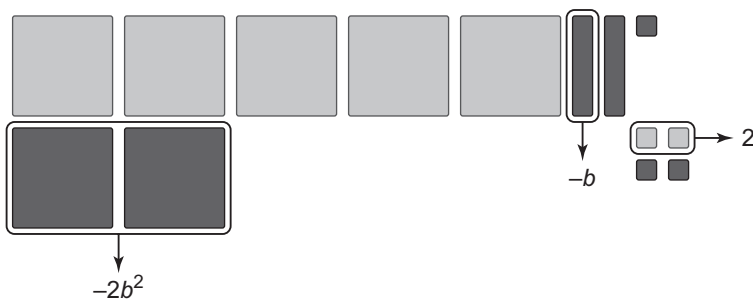
So, add 2 zero pairs of each tile:



So, these tiles also model  $3b^2 - 2b - 1$ .



Take away the tiles for  $-2b^2 - b + 2$ .



The remaining tiles represent:  $5b^2 - b - 3$

The subtraction sentence is:  $(3b^2 - 2b - 1) - (-2b^2 - b + 2) = 5b^2 - b - 3$

## Check

---

1. Use algebra tiles to model each difference.

Sketch the tiles that remain, then write the difference.

**a)**  $(4p + 3) - (2p + 1)$

Remaining tiles: \_\_\_\_\_

So,  $(4p + 3) - (2p + 1) =$  \_\_\_\_\_

**b)**  $(5t + 1) - (-2t + 3)$

Remaining tiles: \_\_\_\_\_

So,  $(5t + 1) - (-2t + 3) =$  \_\_\_\_\_

**c)**  $(3e^2 + 2e - 4) - (4e^2 + 3e - 2)$

Remaining tiles: \_\_\_\_\_

So,  $(3e^2 + 2e - 4) - (4e^2 + 3e - 2) =$  \_\_\_\_\_

---

*Remember to add zero pairs when there are not enough tiles to subtract.*

To subtract integers without tiles, we can add the opposite integer.

To subtract polynomials without tiles, we can add the opposite terms.

### **Example 2** Subtracting Polynomials Symbolically

Subtract:  $(-5k^2 + 2k - 6) - (3k^2 - 4k + 1)$

#### **Solution**

$$\begin{aligned} &(-5k^2 + 2k - 6) - (3k^2 - 4k + 1) \\ &= -5k^2 + 2k - 6 - (3k^2 - 4k + 1) \\ &= -5k^2 + 2k - 6 + (-3k^2 + 4k - 1) \\ &= -5k^2 + 2k - 6 - 3k^2 + 4k - 1 \\ &= -5k^2 - 3k^2 + 2k + 4k - 6 - 1 \\ &= -8k^2 + 6k - 7 \end{aligned}$$

Remove the brackets from the first term.  
Add the opposite of each term in brackets.  
Remove the brackets.  
Group like terms.  
Add the coefficients of like terms.

## Check

### 1. Subtract.

**a)**  $(8f - 3) - (7f + 5)$   
 $=$  \_\_\_\_\_  $- (7f + 5)$

$= 8f - 3 +$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

Remove the brackets from the first term.

The opposite of  $7f$  is: \_\_\_\_\_

The opposite of 5 is: \_\_\_\_\_

Add the opposites.

Remove the brackets.

Group like terms.

Add the coefficients of like terms.

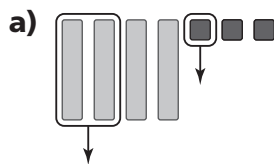
**b)**  $(2 + 5g - 7g^2) - (9g - 4g^2 + 2)$

$=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

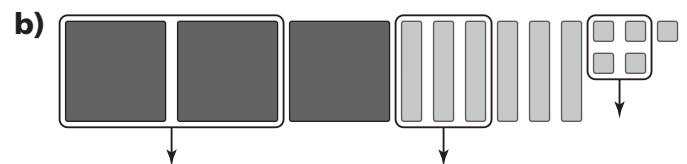
*Remember to write the polynomial in descending order.*

## Practice

### 1. Write the subtraction sentence modelled by each set of tiles.



\_\_\_\_\_  
 \_\_\_\_\_



\_\_\_\_\_  
 \_\_\_\_\_

### 2. Use algebra tiles to model each difference.

Sketch the tiles that remain, then write the difference.

**a)**  $(3r + 2) - (-2r + 3)$

Remaining tiles: \_\_\_\_\_  
 So,  $(3r + 2) - (-2r + 3) =$  \_\_\_\_\_

**b)**  $(-4v^2 + 5v - 1) - (-3v^2 + 4v - 2)$

Remaining tiles: \_\_\_\_\_  
 So,  $(-4v^2 + 5v - 1) - (-3v^2 + 4v - 2)$   
 $=$  \_\_\_\_\_

3. Write the opposite of each term.

**a)**  $-9$ : \_\_\_\_\_      **b)**  $3r$ : \_\_\_\_\_      **c)**  $-2s^2$ : \_\_\_\_\_      **d)**  $t$ : \_\_\_\_\_

4. Subtract.

**a)**  $(4p + 1) - (p + 10)$   
= \_\_\_\_\_  $- (p + 10)$   
  
=  $4p + 1 +$  \_\_\_\_\_  
= \_\_\_\_\_  
= \_\_\_\_\_  
= \_\_\_\_\_

Remove the brackets from the first term.  
The opposite of  $p$  is: \_\_\_\_\_  
The opposite of  $10$  is: \_\_\_\_\_  
Add the opposites.  
Remove the brackets.  
Group like terms.  
Add the coefficients of like terms.

**b)**  $(3h^2 + 5h - 4) - (h^2 - 4h + 6)$   
= \_\_\_\_\_  
= \_\_\_\_\_  
= \_\_\_\_\_  
= \_\_\_\_\_  
= \_\_\_\_\_

Remove the brackets from the first term.  
Add the opposites.  
Remove the brackets.  
Group like terms.  
Add the coefficients of like terms.

**c)**  $(4q^2 + 3) - (3q - q^2 + 3)$   
= \_\_\_\_\_  
= \_\_\_\_\_  
= \_\_\_\_\_  
= \_\_\_\_\_  
= \_\_\_\_\_

5. Check each solution. Identify any errors and correct them.

**a)**  $(7x^2 + 3x + 7) - (3x^2 - 4)$   
=  $7x^2 + 3x + 7 - 3x^2 - 4$   
=  $7x^2 - 3x^2 + 3x + 7 - 4$   
=  $4x^2 + 3x + 3$

$(7x^2 + 3x + 7) - (3x^2 - 4)$   
= \_\_\_\_\_  
= \_\_\_\_\_  
= \_\_\_\_\_

**b)**  $(3a^2 - 2a + 4) - (2a^2 + 3)$   
=  $3a^2 - 2a + 4 - 2a^2 - 3$   
=  $3a^2 - 2a^2 - 2a + 4 - 3$   
=  $a^2 + 2a - 3$

$(3a^2 - 2a + 4) - (2a^2 + 3)$   
= \_\_\_\_\_  
= \_\_\_\_\_  
= \_\_\_\_\_





### Can you ...

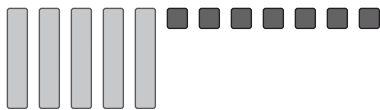
- Recognize, write, describe, and classify polynomials?
- Represent polynomials using tiles, pictures, and algebraic expressions?
- Simplify polynomials by combining like terms?
- Add and subtract polynomials?

**5.1** 1. Is the polynomial a monomial, binomial, or trinomial?

- a)  $-9$                                       The polynomial has \_\_\_\_ term, so it is a \_\_\_\_\_.
- b)  $3f - 5$                                     The polynomial has \_\_\_\_ terms, so it is a \_\_\_\_\_.
- c)  $2s^2 - s + 1$                             The polynomial has \_\_\_\_ terms, so it is a \_\_\_\_\_.
- d)  $-a^2 + 2a$                                 The polynomial has \_\_\_\_ terms, so it is a \_\_\_\_\_.

2. Write the polynomial represented by each set of tiles.

a) Use the variable  $g$ .



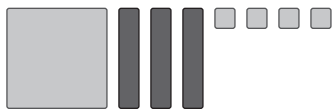
\_\_\_\_\_

b) Use the variable  $r$ .



\_\_\_\_\_

c) Use the variable  $w$ .



\_\_\_\_\_

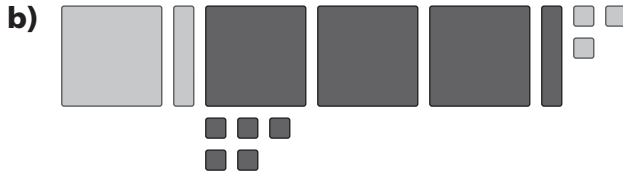
**5.2** 3. Simplify each tile model.

Write the polynomial that the remaining tiles represent.



Remaining tiles: \_\_\_\_\_

Polynomial: \_\_\_\_\_



Remaining tiles: \_\_\_\_\_ Polynomial: \_\_\_\_\_

**4.** Simplify each polynomial.

**a)**  $8e - 9 - 5e + 4$       Group like terms.  
 = \_\_\_\_\_      Add the coefficients of like terms.  
 = \_\_\_\_\_

**b)**  $4d^2 - 3d + 11 - d^2 + 5d - 13$   
 = \_\_\_\_\_  
 = \_\_\_\_\_

**5.3** **5.** Sketch tiles to model each sum.  
 Then write the sum.

**a)**  $(4v - 4) + (-2v + 7)$

Remaining tiles: \_\_\_\_\_  
 So,  $(4v - 4) + (-2v + 7) =$  \_\_\_\_\_

**b)**  $(6u^2 - 5u - 7) + (-3u^2 + 3u + 7)$

Remaining tiles: \_\_\_\_\_  
 So,  $(6u^2 - 5u - 7) + (-3u^2 + 3u + 7) =$  \_\_\_\_\_

**6.** Add.

**a)**  $(3t + 11) + (-7t - 4)$

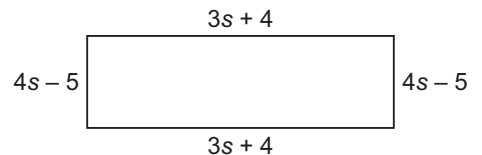
$$\begin{array}{r} 3t + 11 \\ + -7t - 4 \\ \hline \quad t + \quad \end{array}$$

**b)**  $(10y^2 - 9) + (-3y^2 + 4y - 2)$

$$\begin{array}{r} 10y^2 \quad - 9 \\ + -3y^2 + 4y - 2 \\ \hline \end{array}$$

**7.** Find the perimeter of this rectangle.

Perimeter = \_\_\_\_\_  
 = \_\_\_\_\_  
 = \_\_\_\_\_  
 = \_\_\_\_\_



**5.4** 8. Use algebra tiles to model each difference.

Sketch the tiles that remain, then write the difference.

**a)**  $(5n - 6) - (-n - 3)$

Remaining tiles: \_\_\_\_\_

So,  $(5n - 6) - (-n - 3) =$  \_\_\_\_\_

**b)**  $(-v^2 + 3v - 5) - (-v^2 + 4v + 2)$

Remaining tiles: \_\_\_\_\_

So,  $(-v^2 + 3v - 5) - (-v^2 + 4v + 2) =$  \_\_\_\_\_

**9.** Subtract.

**a)**  $(11h + 3) - (9h - 2)$

$=$  \_\_\_\_\_  $- (9h - 2)$

$=$  \_\_\_\_\_  $+ ($  \_\_\_\_\_  $)$

$=$  \_\_\_\_\_

$=$  \_\_\_\_\_

$=$  \_\_\_\_\_

Remove the brackets from the first term.

Add the opposites.

Remove the brackets.

Group like terms.

Add the coefficients of like terms.

**b)**  $(7j^2 - 11j - 7) - (12j^2 - 8j - 3)$

$=$  \_\_\_\_\_

$=$  \_\_\_\_\_

$=$  \_\_\_\_\_

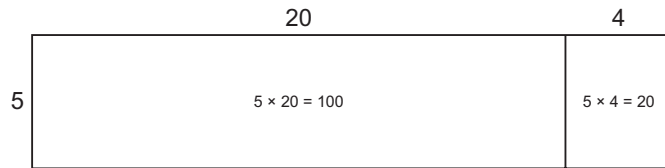
$=$  \_\_\_\_\_

$=$  \_\_\_\_\_

## 5.5 Skill Builder

### The Distributive Property

We can use this diagram to model  $5 \times 24$ .



This diagram shows:

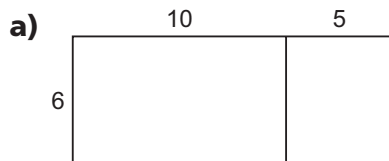
$$\begin{aligned} 5 \times 24 &= 5 \times (20 + 4) \\ &= (5 \times 20) + (5 \times 4) \\ &= 100 + 20 \\ &= 120 \end{aligned}$$

*We multiply the term outside the brackets by each term inside the brackets, then find the sum.*

This shows the **distributive property** of multiplication.

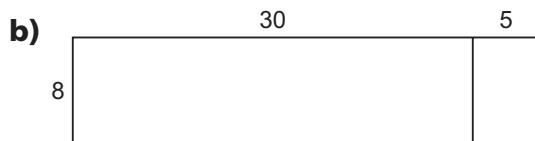
### Check

1. How does each diagram show the distributive property?



\_\_\_\_\_

\_\_\_\_\_



\_\_\_\_\_

2. Use the distributive property to multiply.

a)  $7 \times 21 = 7 \times (20 + 1)$   
 $= (7 \times 20) + (7 \times 1)$   
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

b)  $8 \times 43 = 8 \times (40 + 3)$   
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

## Multiplying and Dividing Integers

When multiplying or dividing 2 integers, look at the sign of each integer:

- When the integers have the same sign, their product or quotient is positive.
- When the integers have different signs, their product or quotient is negative.

$\times/\div$	(-)	(+)
(-)	(+)	(-)
(+)	(-)	(+)

$$7 \times (-4)$$

$$7 \times (-4) = -28$$

These 2 integers have different signs, so their product is negative.

$$(-12) \div (-3)$$

$$(-12) \div (-3) = 4$$

These 2 integers have the same sign, so their quotient is positive.

*When one number is divided by another number, the result is called the quotient.*

### Check

1. Will the product be positive or negative?

a)  $9 \times 5$  \_\_\_\_\_

b)  $8 \times (-3)$  \_\_\_\_\_

c)  $(-12) \times 5$  \_\_\_\_\_

d)  $(-7) \times (-6)$  \_\_\_\_\_

2. Multiply.

a)  $6 \times 5 =$  \_\_\_\_\_

b)  $4 \times (-10) =$  \_\_\_\_\_

c)  $(-7) \times 3 =$  \_\_\_\_\_

d)  $(-8) \times (-6) =$  \_\_\_\_\_

e)  $12 \times (-5) =$  \_\_\_\_\_

f)  $(-4) \times (-8) =$  \_\_\_\_\_

3. Will the quotient be positive or negative?

a)  $18 \div 3$  \_\_\_\_\_

b)  $(-36) \div 6$  \_\_\_\_\_

c)  $72 \div (-9)$  \_\_\_\_\_

d)  $(-48) \div (-8)$  \_\_\_\_\_

4. Divide.

a)  $(-49) \div 7 =$  \_\_\_\_\_

b)  $(-56) \div (-8) =$  \_\_\_\_\_

c)  $48 \div 6 =$  \_\_\_\_\_

d)  $81 \div (-9) =$  \_\_\_\_\_

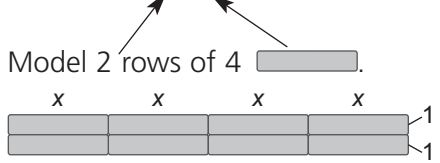
e)  $(-27) \div (-3) =$  \_\_\_\_\_

f)  $(-42) \div 7 =$  \_\_\_\_\_

# 5.5 Multiplying and Dividing a Polynomial by a Constant

**FOCUS** Use different strategies to multiply and divide a polynomial by a constant.

To multiply  $2(4x)$  with algebra tiles:



Recall:  $2(4x) = 2 \times 4x$

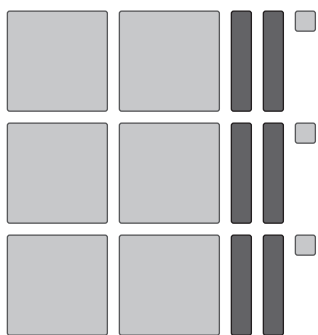
There are 8  $x$ -tiles. So,  $2(4x) = 8x$




## Example 1 Using Algebra Tiles to Multiply a Polynomial by a Constant

Find the product:  $3(2b^2 - 2b + 1)$

### Solution

$3(2b^2 - 2b + 1)$



Model 3 rows of 2 , 2 , and 1 .

These tiles represent:  $6b^2 - 6b + 3$ .

So,  $3(2b^2 - 2b + 1) = 6b^2 - 6b + 3$

### Check

- Sketch algebra tiles to multiply. Write the product each time.
  - $3(4p - 3) =$  \_\_\_\_\_
  - $2(-s^2 + s + 3) =$  \_\_\_\_\_

When working symbolically, remember the rules for integer multiplication and division.

### Example 2 Using the Distributive Property to Multiply a Polynomial by a Constant

Find the product:  $-5(4e^2 - 5e + 3)$

#### Solution

$$\begin{aligned} & -5(4e^2 - 5e + 3) && \text{Multiply each term in brackets by } -5. \\ = & (-5)(4e^2) + (-5)(-5e) + (-5)(3) && \text{Multiply.} \\ = & -20e^2 + 25e + (-15) \\ = & -20e^2 + 25e - 15 \end{aligned}$$

#### Check

1. Multiply.

a)  $3(7s^2 + 9)$  Multiply each term in brackets by 3.  
 $= 3(7s^2) + 3(9)$  Multiply:  $3 \times 7 = \underline{\quad}$  and  $3 \times 9 = \underline{\quad}$   
 $= \underline{\quad} s^2 + \underline{\quad}$

b)  $-4(5e^2 - 8e)$  Multiply each term in brackets by  $-4$ .  
 $= \underline{\hspace{2cm}}$  Multiply.  
 $= \underline{\hspace{2cm}}$

c)  $-5(-2d^2 - 3d + 6)$  d)  $7(6y^2 - 8y + 9)$   
 $= \underline{\hspace{2cm}}$   $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   $= \underline{\hspace{2cm}}$

We can use algebra tiles to divide a polynomial by a constant.

To divide:  $(-8x) \div 2$

Arrange 8  into 2 equal rows.



In each row there are 4 .

So,  $(-8x) \div 2 = -4x$

### Example 3 Using Algebra Tiles to Divide a Polynomial by a Constant

Find the quotient:  $(6s - 9) \div 3$

#### Solution

$$(6s - 9) \div 3$$

Arrange 6  and 9  into 3 equal rows.



In each row, there are 2  and 3 .

$$\text{So, } (6s - 9) \div 3 = 2s - 3$$

#### Check

1. Sketch algebra tiles to divide. Write the quotient each time.

**a)**  $(3g^2 + 12g) \div 3 =$  \_\_\_\_\_      **b)**  $(-4b^2 + 6) \div 2 =$  \_\_\_\_\_

**c)**  $(4s^2 - 4s + 8) \div 4 =$  \_\_\_\_\_      **d)**  $(-6t^2 + 9t - 9) \div 3 =$  \_\_\_\_\_



When algebra tiles are not available,  
or when the divisor is negative,  
we can use what we already know about division.

*In the division sentence  
 $6 \div 3 = 2$ , the divisor is 3.*

We can write  $8x \div 4$  as a fraction:  $\frac{8x}{4}$

We write the fraction as a product, then simplify each fraction.

$$\begin{aligned} \frac{8x}{4} &= \frac{8}{4} \times x \\ &= 2 \times x \\ &= 2x \end{aligned}$$

### **Example 4** Dividing a Polynomial by a Constant Symbolically

Find the quotient:  $\frac{-9v^2 + 6}{3}$

#### **Solution**

$$\frac{-9v^2 + 6}{3}$$

Write as the sum of 2 fractions with denominator 3.

$$= \frac{-9v^2}{3} + \frac{6}{3}$$

Simplify the fractions.

$$= \frac{-9}{3} \times v^2 + 2$$

When 2 integers have different signs, the quotient is negative.

$$= -3 \times v^2 + 2$$

$$= -3v^2 + 2$$

### **Check**

1. Divide.

a)  $\frac{12r^2 + 8}{4}$

Write as the sum of 2 fractions with denominator 4.

$$= \frac{\quad}{4} + \frac{\quad}{4}$$

Simplify the fractions.

$$= \quad \times r^2 + \quad$$

When 2 integers have the same sign, the quotient is \_\_\_\_\_.

$$= \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

**b)**  $\frac{18v^2 - 6v + 12}{6}$

$= \frac{\quad}{6} + \frac{\quad}{6} + \frac{\quad}{6}$

$= \frac{\quad}{6} \times \underline{\quad} + \frac{\quad}{6} \times \underline{\quad} + \underline{\quad}$

$= \underline{\quad}$

$= \underline{\quad}$

**c)**  $\frac{-4e^2 - 8e}{2}$

$= \underline{\quad}$

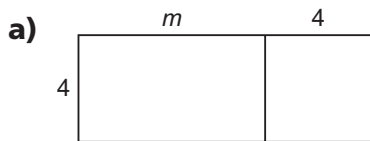
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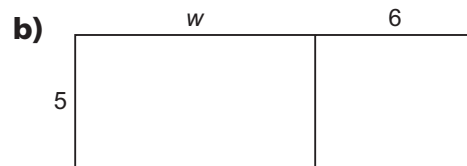
$= \underline{\quad}$

**Practice**

1. Which multiplication sentence does each rectangle represent?



$4(m + 4) = (4 \times \underline{\quad}) + (4 \times \underline{\quad})$   
 $= \underline{\quad}$



$\underline{\quad}$   
 $= \underline{\quad}$

2. Write the multiplication sentence modelled by each set of tiles.



$\underline{\quad}$



$\underline{\quad}$

3. Sketch algebra tiles to multiply. Write the product each time.

**a)**  $3(6r - 4) = \underline{\quad}$

**b)**  $2(-2b^2 - b + 3) = \underline{\quad}$

**4.** Multiply.

**a)**  $6(-4t^2 + 3)$   
 $= 6(\underline{\quad}) + 6(\underline{\quad})$   
 $= \underline{\hspace{2cm}}$

**b)**  $-8(-3k^2 - 2k + 4)$   
 $= \underline{\hspace{2cm}}$   
 $\underline{\hspace{2cm}}$   
 $\underline{\hspace{2cm}}$

**5.** Which of these quotients is modelled by the tiles below?



**a)**  $(15x - 9) \div 3$

**b)**  $(-15x - 9) \div 3$

**c)**  $(-15x + 9) \div 3$

**6.** Sketch algebra tiles to divide. Write the quotient each time.

**a)**  $(3h^2 - 15h) \div 3 = \underline{\hspace{2cm}}$

**b)**  $(-2a^2 - 6a + 4) \div 2 = \underline{\hspace{2cm}}$

**7.** Divide.

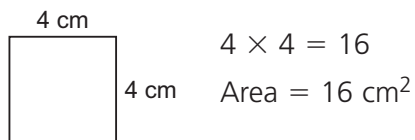
**a)**  $\frac{-10z^2 + 15}{5}$   
 $= \frac{\underline{\hspace{1cm}}}{5} + \frac{\underline{\hspace{1cm}}}{5}$   
 $= \underline{\hspace{1cm}} \times z^2 + \underline{\hspace{1cm}}$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

**b)**  $\frac{7x^2 - 7x + 21}{-7}$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

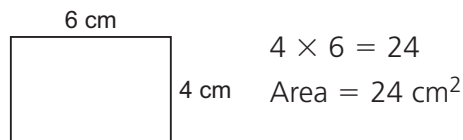
## 5.6 Skill Builder

### Multiplying Monomials

The area of this square is:

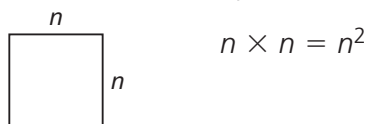


The area of this rectangle is:

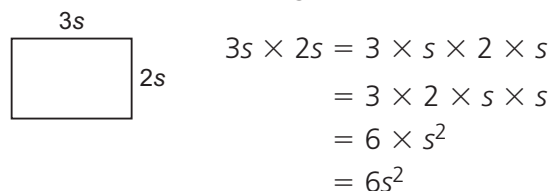


We can use the models above to help us multiply 2 monomials.

The area of this square is:



The area of this rectangle is:



When one or both of the monomials is negative, we cannot use an area model.

We multiply using the rules for multiplying integers.

$$\begin{aligned}
 4v \times (-2v) &= 4 \times v \times (-2) \times v \\
 &= 4 \times (-2) \times v \times v \\
 &= -8 \times v^2 \\
 &= -8v^2
 \end{aligned}$$

*4 and -2 have different signs,  
so their product is negative.*

### Check

1. Multiply.

a)  $b \times b = \underline{\hspace{2cm}}$

b)  $c \times (-c) = \underline{\hspace{2cm}}$

c)  $(-f) \times (-f) = \underline{\hspace{2cm}}$

d)  $(-g) \times g = \underline{\hspace{2cm}}$

2. Multiply.

a)  $5r \times 6r = 5 \times r \times 6 \times r$   
 $= 5 \times 6 \times r \times r$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

b)  $(-2d) \times 8d = (-2) \times d \times 8 \times d$   
 $= (-2) \times 8 \times d \times d$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

c)  $4a \times (-7a) = \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

d)  $(-5v) \times (-9v) = (-5) \times v \times (-9) \times v$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

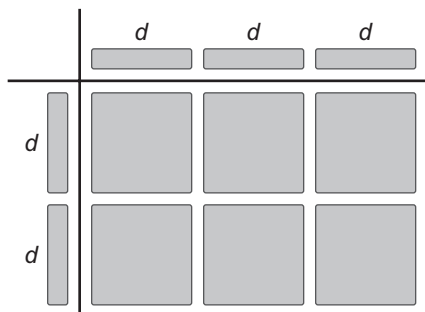
## 5.6 Multiplying and Dividing a Polynomial by a Monomial

**FOCUS** Use different strategies to multiply and divide a polynomial by a monomial.

To multiply  $2d(3d)$  with algebra tiles:

Draw 2 adjacent sides of a rectangle.

Position  tiles to show side lengths  $2d$  and  $3d$ .



$d \times d = d^2$ , so use a  $d^2$ -tile.

Then fill the rectangle with tiles.

We used 6  $d^2$ -tiles to fill the rectangle. So,  $2d(3d) = 6d^2$

### Example 1 Multiplying a Binomial by a Monomial

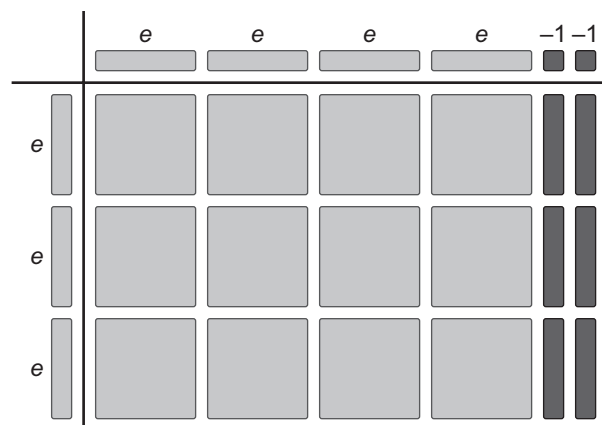
Find the product:  $3e(4e - 2)$

#### Solution

$3e(4e - 2)$

Draw 2 adjacent sides of a rectangle.

Position tiles to show side lengths  $3e$  and  $4e - 2$ .



When two tiles have the same colour, use a positive tile in the rectangle.

When two tiles have different colours, use a negative tile in the rectangle.

Fill the rectangle with tiles.

We used 12  $e^2$ -tiles and 6  $-e$ -tiles to fill the rectangle.

So,  $3e(4e - 2) = 12e^2 - 6e$

## Check

---

1. Sketch algebra tiles to multiply. Write the product each time.

**a)**  $2f(4f)$

**b)**  $3m(-2m + 4)$

Number of  $f^2$ -tiles used: \_\_\_\_\_

So,  $2f(4f) = \underline{\hspace{2cm}} f^2$

$3m(-2m + 4) = \underline{\hspace{4cm}}$

---

### **Example 2** Multiplying a Binomial by a Monomial Symbolically

Find each product:

**a)**  $3x(9x - 4)$

**b)**  $-6x(-7x + 5)$

#### **Solution**

---

**a)**  $3x(9x - 4)$

$= (3x)(9x) + (3x)(-4)$

$= 27x^2 + (-12x)$

$= 27x^2 - 12x$

Use the distributive property.

Multiply each term in brackets by  $3x$ .

Multiply:  $3 \times 9 = 27$  and  $3(-4) = -12$

**b)**  $-6x(-7x + 5)$

$= (-6x)(-7x) + (-6x)(5)$

$= 42x^2 + (-30x)$

$= 42x^2 - 30x$

Multiply each term in brackets by  $-6x$ .

Multiply:  $(-6)(-7) = 42$  and  $(-6)(5) = -30$

## Check

1. Multiply.

$$\begin{aligned}\text{a) } & 7x(4x + 5) \\ & = (7x)(4x) + (7x)(5) \\ & = \underline{\quad}x^2 + \underline{\quad}x\end{aligned}$$

Multiply each term in brackets by 7x.  
Multiply:  $7 \times 4 = \underline{\quad}$  and  $7 \times 5 = \underline{\quad}$

$$\begin{aligned}\text{b) } & s(-3s + 4) \\ & = (s)(-3s) + (s)(4) \\ & = \underline{\hspace{2cm}}\end{aligned}$$

Multiply each term in brackets by s.  
Multiply:  $1 \times (-3) = \underline{\quad}$  and  $1 \times 4 = \underline{\quad}$

$$\begin{aligned}\text{c) } & -9r(4r - 5) \\ & = \underline{\hspace{2cm}} \\ & = \underline{\hspace{2cm}}\end{aligned}$$

To divide a polynomial by a monomial, we use what we already know about division.

To divide:  $\frac{6a^2}{3a}$

We write the fraction as a product of 2 fractions, then simplify each fraction.

$$\begin{aligned}\frac{6a^2}{3a} & = \frac{6}{3} \times \frac{a^2}{a} \\ & = 2 \times \frac{a \times \cancel{a}^1}{\cancel{a}_1} \\ & = 2 \times a \\ & = 2a\end{aligned}$$

*a is a common factor of the numerator and the denominator.*

2a is the quotient of  $\frac{6a^2}{3a}$ .

### Example 3 Dividing a Binomial by a Monomial

Find the quotient:  $\frac{-8s^2 + 6s}{-2s}$

#### Solution

$$\begin{aligned}& \frac{-8s^2 + 6s}{-2s} \\ & = \frac{-8s^2}{-2s} + \frac{6s}{-2s} \\ & = \frac{-8}{-2} \times \frac{s^2}{s} + \frac{6}{-2} \times \frac{s}{s} \\ & = 4 \times s + (-3) \times 1 \\ & = 4s - 3\end{aligned}$$

Write as the sum of 2 fractions each with denominator  $-2s$ .

Simplify the fractions.

*A variable divided by itself is 1.*

## Check

1. Divide.

a)  $\frac{12a^2}{-6a}$

=  $\frac{\quad}{\quad} \times \frac{a^2}{a}$

=  $\frac{\quad}{\quad} \times \frac{a \times \cancel{a}^1}{\cancel{a}^1}$

=  $\frac{\quad}{\quad}$

=  $\frac{\quad}{\quad}$

Write as a product of 2 fractions.

Simplify each fraction.

b)  $\frac{9b^2 + 3b}{3b}$

=  $\frac{\quad}{3b} + \frac{\quad}{3b}$

=  $\frac{\quad}{\quad}$

=  $\frac{\quad}{\quad}$

=  $\frac{\quad}{\quad}$

c)  $\frac{-14c^2 + 21c}{-7c}$

=  $\frac{\quad}{\quad}$

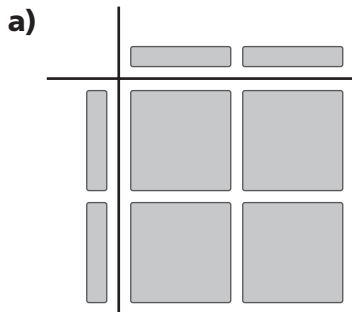
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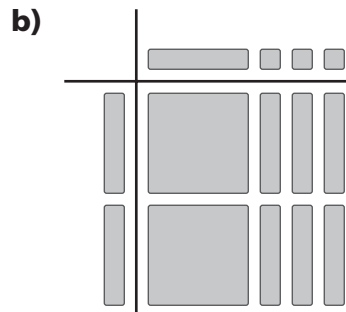
=  $\frac{\quad}{\quad}$

## Practice

1. Write the multiplication sentence modelled by each set of tiles.



\_\_\_\_\_



\_\_\_\_\_



2. Sketch algebra tiles to multiply. Write the product each time.

a)  $2s(s + 4) =$  \_\_\_\_\_

b)  $t(-2t + 3) =$  \_\_\_\_\_

3. Multiply.

a)  $4r(5r - 1)$   
 $= (4r)(\underline{\quad}) + (4r)(\underline{\quad})$   
 $= \underline{\quad}r^2 + (\underline{\quad}r)$   
 $=$  \_\_\_\_\_

b)  $7s(-3s + 6)$   
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

c)  $-6t(t - 3)$   
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

d)  $-8u(-6u + 7)$   
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

4. Divide.

a)  $\frac{12v^2}{4v}$   
 $=$  \_\_\_\_\_  $\times \frac{v^2}{v}$   
 $=$  \_\_\_\_\_  $\times \frac{\cancel{v} \times \cancel{v}^1}{\cancel{v}_1}$   
 $=$  \_\_\_\_\_  $\times v$   
 $=$  \_\_\_\_\_

b)  $\frac{15w^2}{-3w}$   
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

c)  $\frac{-28x^2}{-7x}$   
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

5. Divide.

a)  $\frac{18y^2 + 12y}{2y}$   
 $= \frac{\quad}{2y} + \frac{\quad}{2y}$   
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

b)  $\frac{-32z^2 + 24z}{-8z}$   
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

c)  $\frac{15n^2 + 21n}{-3n}$   
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_  
 $=$  \_\_\_\_\_

# Unit 5 Puzzle

## Alphabet Soup!

The table below contains 15 polynomial expressions.  
Simplify each expression.

<b>1</b> $2x(x - 3)$ _____	<b>4</b> $(3x + 7) - (11 - 4x)$ _____	<b>3</b> $\frac{5x^2 + 10x}{5x}$ _____
<b>6</b> $-4(x^2 + 3x - 1)$ _____	<b>5</b> $(5x + 4) + (x^2 - 2x + 1)$ _____	<b>7</b> $3(2x - 1)$ _____
<b>9</b> $3x^2 + 5 - 2x$ $- (5 + 3x^2 - 2x)$ _____	<b>10</b> $8 + 7x - 11 - 3x$ _____	<b>13</b> $(4x^2 + 9x + 5)$ $- (4x^2 + 8x + 3)$ _____
<b>8</b> $\frac{36x^2 - 18x}{6x}$ _____	<b>2</b> $2x^2 + x + 5 - 7x - 5$ _____	<b>11</b> $(16x - 12) \div 4$ _____
<b>0</b> $(6x + 5) + (-9 + x)$ _____	<b>12</b> $2(-2x^2 - 6x + 2)$ _____	<b>15</b> $(4x^2 + x + 6)$ $- (3x^2 - 2x + 1)$ _____

Seven pairs of expressions have the same answer. Find the 7 pairs.

For each matching pair, add the numbers in the top left corner of each square.

The sum represents a letter of the alphabet.

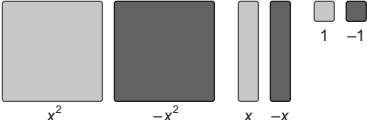
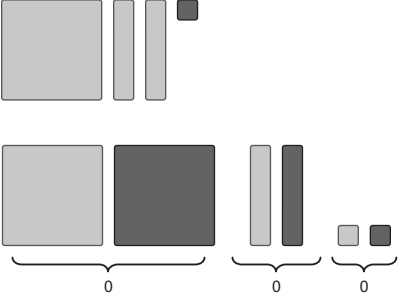
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Write the seven letters below.

\_\_\_\_\_

Unscramble the letters to find a math word used in this unit. \_\_\_\_\_

# Unit 5 Study Guide

Skill	Description	Example
Recognize the different parts of a polynomial.	A polynomial may have variable terms and a constant term. The number in front of a variable is its coefficient.	<p>variable term</p> $\textcircled{3x^2} + 2x + 4$ <p>coefficient      constant</p>
Describe and classify polynomials.	A polynomial can be classified by its number of terms and by its term with the greatest degree.	<p>Monomial: <math>3x</math></p> <p>Binomial: <math>2x + 5</math></p> <p>Trinomial: <math>x^2 + 2x - 1</math> degree 2</p>
Use algebra tiles to represent a polynomial.	<p>We use these tiles:</p>  <p><math>x^2</math>    <math>-x^2</math>    <math>x</math>    <math>-x</math></p> <p>A pair of tiles with the same shape and size, but different colours forms a zero pair. The tiles model 0.</p>	<p><math>x^2 + 2x - 1</math></p>  <p>0      0      0</p>
Simplify polynomials by combining like terms.	To simplify a polynomial, add the coefficients of like terms.	<p>Like terms: <math>4x^2</math> and <math>-2x^2</math></p> <p>Unlike terms: <math>3x</math> and <math>-5</math></p> $4x^2 - 2x^2 = 2x^2$
Add polynomials.	To add polynomials, remove the brackets and add the coefficients of like terms.	$\begin{aligned} &(4x^2 + 3x) + (x^2 - 5x) \\ &= 4x^2 + 3x + x^2 - 5x \\ &= 4x^2 + x^2 + 3x - 5x \\ &= 5x^2 - 2x \end{aligned}$
Subtract polynomials.	To subtract a polynomial, add the opposite terms.	$\begin{aligned} &(3x^2 + 5x) - (2x^2 - x) \\ &= 3x^2 + 5x + (-2x^2 + x) \\ &= 3x^2 + 5x - 2x^2 + x \\ &= 3x^2 - 2x^2 + 5x + x \\ &= x^2 + 6x \end{aligned}$
Multiply a polynomial by a monomial.	To multiply a polynomial by a monomial, use the distributive property.	$\begin{aligned} &3x(6x - 5) \\ &= 3x(6x) + (3x)(-5) \\ &= 18x^2 + (-15x) \\ &= 18x^2 - 15x \end{aligned}$
Divide a polynomial by a monomial.	To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.	$\frac{24x^2 - 32x}{8x} = \frac{24x^2}{8x} + \frac{-32x}{8x}$ $= 3x - 4$

# Unit 5 Review

**5.1** 1. Is the polynomial a monomial, binomial, or trinomial?

a)  $-3s^2 + 11$  \_\_\_\_\_.

b)  $8d$  \_\_\_\_\_.

c)  $2e^2 - 9e + 7$  \_\_\_\_\_.

d)  $8h - 1$  \_\_\_\_\_.

2. Sketch algebra tiles to model each polynomial.

a)  $3k - 4$

b)  $2m^2 - m + 3$

c)  $-n^2 + 5n - 2$

\_\_\_\_\_

**5.2** 3. Simplify each polynomial.

a)  $-7d - 4 + 8d + 2$

= \_\_\_\_\_  
= \_\_\_\_\_

b)  $3e^2 - 8e + 2e^2 + 11e$

= \_\_\_\_\_  
= \_\_\_\_\_

c)  $13 - 6h + 2h^2 + 7h - 9$

= \_\_\_\_\_  
= \_\_\_\_\_  
= \_\_\_\_\_

d)  $-9k^2 + 15k - 8 - 2k^2 - 4k + 3$

= \_\_\_\_\_  
= \_\_\_\_\_

4. Identify and explain any errors you find.

a)  $2x^2 + 5x = 7x^2$

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

b)  $5s - 7s = -2s$

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**5.3** 5. Sketch algebra tiles to model each sum. Then write the sum.

a)  $(-5e + 7) + (4e - 1)$

b)  $(6f^2 - 2f + 5) + (-4f^2 - f - 3)$

Remaining tiles: \_\_\_\_\_  
So,  $(-5e + 7) + (4e - 1) =$  \_\_\_\_\_

Remaining tiles: \_\_\_\_\_  
So,  $(6f^2 - 2f + 5) + (-4f^2 - f - 3)$   
= \_\_\_\_\_

6. Add.

**a)**  $(7r + 11) + (-2r + 3)$   
 = \_\_\_\_\_  
 = \_\_\_\_\_  
 = \_\_\_\_\_

**b)**  $(-9s^2 + 5s) + (16s^2 - 9s - 14)$   
 = \_\_\_\_\_  
 = \_\_\_\_\_  
 = \_\_\_\_\_

**5.4** 7. Use algebra tiles to model each difference.

Sketch the tiles that remain, then write the difference.

**a)**  $(-2t + 5) - (-5t + 7)$

Remaining tiles: \_\_\_\_\_  
 So,  $(-2t + 5) - (-5t + 7) =$  \_\_\_\_\_

**b)**  $(-7u - 2) - (-u^2 - 3u - 1)$

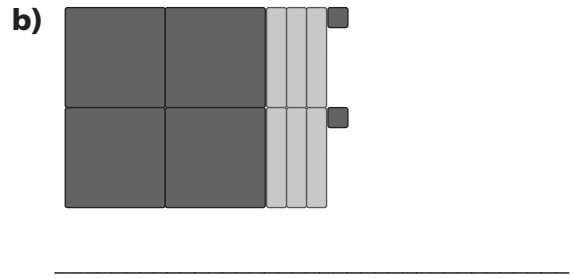
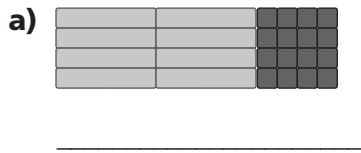
Remaining tiles: \_\_\_\_\_  
 So,  $(-7u - 2) - (-u^2 - 3u - 1) =$  \_\_\_\_\_

8. Subtract.

**a)**  $(6v + 5) - (13v - 3)$   
 =  $6v + 5 +$  (\_\_\_\_\_)  
 = \_\_\_\_\_  
 = \_\_\_\_\_  
 = \_\_\_\_\_

**b)**  $(10w^2 - 7) - (-2w + 9w^2 + 5)$   
 = \_\_\_\_\_  
 = \_\_\_\_\_  
 = \_\_\_\_\_  
 = \_\_\_\_\_

**5.5** 9. Write the multiplication sentence modelled by each set of tiles.



10. Multiply.

**a)**  $6(-7y^2 + 1)$   
 =  $6$ (\_\_\_\_\_) +  $6$ (\_\_\_\_\_)  
 = \_\_\_\_\_

**b)**  $-9(-2z^2 - 4z + 5)$   
 = \_\_\_\_\_  
 = \_\_\_\_\_  
 = \_\_\_\_\_

11. Divide.

a)  $\frac{16a-40}{8}$

=  $\frac{\quad}{8} + \frac{\quad}{8}$

=  $\frac{16}{8} \times a + (\quad)$

= \_\_\_\_\_

= \_\_\_\_\_

b)  $\frac{27b^2 - 9b + 36}{-9}$

= \_\_\_\_\_

= \_\_\_\_\_

= \_\_\_\_\_

= \_\_\_\_\_

5.6 12. Sketch algebra tiles to multiply. Write the product each time.

a)  $2c(c + 5) = \underline{\hspace{2cm}}$

b)  $3d(-d + 4) = \underline{\hspace{2cm}}$

13. Multiply.

a)  $3e(5e - 2)$

=  $(3e)(\quad) + (3e)(\quad)$

=  $\underline{\hspace{1cm}}e^2 + (\underline{\hspace{1cm}})e$

= \_\_\_\_\_

b)  $-4f(5f + 2)$

= \_\_\_\_\_

= \_\_\_\_\_

= \_\_\_\_\_

14. Divide.

a)  $\frac{-21k^2}{7k}$

=  $\frac{-21}{7} \times \frac{k^2}{k}$

=  $\underline{\hspace{1cm}} \times \frac{k \times k^1}{k^1}$

=  $\underline{\hspace{1cm}} \times k$

= \_\_\_\_\_

b)  $\frac{81m^2 - 45m}{-9m}$

= \_\_\_\_\_

= \_\_\_\_\_

= \_\_\_\_\_

= \_\_\_\_\_

c)  $\frac{-33n^2 + 36n}{-3n}$

= \_\_\_\_\_

= \_\_\_\_\_

= \_\_\_\_\_

= \_\_\_\_\_