UNIT 7

Similarity and Transformations

What You'll Learn

- Draw and interpret scale diagrams.
- Apply properties of similar polygons.
- Identify and describe line symmetry and rotational symmetry.

Why It's Important

Similarity and scale diagrams are used by

- construction workers when they construct buildings and bridges
- motorists when they use maps to get around a city

Symmetry is used by

- interior designers when they arrange furniture and accessories in a room

Key Words

enlargement	congruent
reduction	reflection
scale diagram	line of reflection
scale factor	tessellation
polygon	rotation
non-polygon	rotational symmetry
similar polygons	order of rotation
proportional	angle of rotation symmetry
line symmetry	translation

7.1 Skill Builder

Converting Between Metric Units of Length

This table shows the relationships among some of the units of length.

1 m = 100 cm 1 m = 1000 mm
1 cm = 0.01 m 1 cm = 10 mm
1 mm = 0.001 m 1 mm = 0.1 cm

To convert 2.3 m to centimetres:
1 m = 100 cm
So, to convert metres to centimetres, multiply by 100.
2.3 m = 2.3(100 cm)
= 230 cm

To convert 255 cm to metres:
1 cm = 0.01 m
So, to convert centimetres to metres, multiply by 0.01.
255 cm = 255(0.01 m)
= 2.55 m

Check

1. Convert each measure to centimetres.

a) 7 m
1 m = ______ cm
So, 7 m = 7(__________)
= __________

b) 21 mm
1 mm = __________
So, 21 mm = _______________
= _______________

2. Convert each measure to metres.

a) 346 cm
1 cm = _______ m
So, 346 cm = 346(__________)
= __________

b) 1800 mm
1 mm = ___________
So, 1800 mm = __________________
= __________________

3. Convert each measure to millimetres.

a) 6.5 cm
1 cm = _____ mm
So, 6.5 cm = 6.5(__________)
= __________

b) 3.8 m
1 m = ____________
So, 3.8 m = __________________
= __________________

7.1 Scale Diagrams and Enlargements

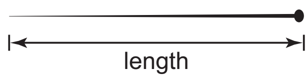
FOCUS **Draw and interpret scale diagrams that represent enlargements.**

A diagram that is an **enlargement** or a **reduction** of another diagram is called a **scale diagram.** The **scale factor** is the relationship between the matching lengths on the two diagrams.

> To find the scale factor of a scale diagram, we divide:
>
> $\dfrac{\text{length on scale diagram}}{\text{length on original diagram}}$

Example 1 Using Matching Lengths to Determine the Scale Factor

Here is a scale diagram of a pin.
The actual length of the pin is 13 mm.
Find the scale factor of the diagram.



Solution

Measure the length of the pin in the diagram.
The length is 3.9 cm, or 39 mm.

The scale factor is: $\dfrac{\text{length on scale diagram}}{\text{length of pin}} = \dfrac{39 \text{ mm}}{13 \text{ mm}}$

$= 3$

The units of length must be the same.

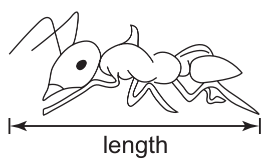
The scale factor is 3.

When the drawing is an enlargement, the scale factor is greater than 1.

Check

1. Find the scale factor for each scale diagram.

a) The actual length of the ant is 6 mm.



Measure the length of the ant in the diagram.
Length = _______ cm, or _______ mm

Scale factor = $\dfrac{\text{length on scale diagram}}{\text{length of ant}}$

$= \dfrac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$

$=$ _______

The scale factor is _______.

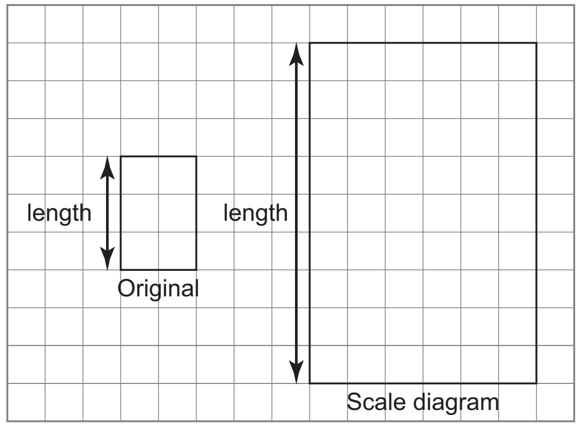
b) Length of rectangle in scale diagram: __________
Length of original rectangle: __________

Scale factor $= \dfrac{\text{length on scale diagram}}{\text{length on original diagram}}$

$= \dfrac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$

$= \_\_\_\_\_\_$

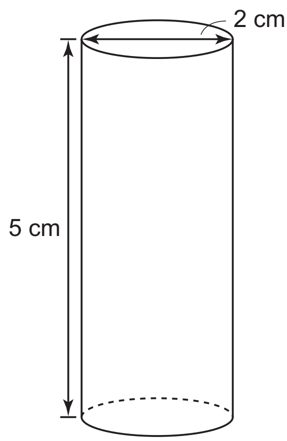
The scale factor is _______.



To find the dimensions of a scale diagram, multiply each length on the original diagram by the scale factor.

Example 2 Using a Scale Factor to Determine Dimensions

This cylinder is to be enlarged by a scale factor of $\frac{5}{2}$.
Find the dimensions of the enlargement.



Solution

Write the scale factor as a decimal.

$\frac{5}{2} = 5 \div 2$
$= 2.5$

To write a fraction as a decimal, divide the numerator by the denominator.

Diameter of original cylinder: 2 cm
Diameter of enlargement: 2.5×2 cm $= 5$ cm

Height of original cylinder: 5 cm
Height of enlargement: 2.5×5 cm $= 12.5$ cm

The enlargement has diameter 5 cm and height 12.5 cm.

Check

1. A photo has dimensions 10 cm by 15 cm.
Enlargements are to be made with each scale factor below.
Find the dimensions of each enlargement.

a) Scale factor 4

The length of a rectangle is always the longer dimension.

Length of original photo: _________
Length of enlargement: $4 \times$ _________ $=$ _________

Width of original photo: _________
Width of enlargement: $4 \times$ _________ $=$ _________

The enlargement has dimensions _________________________.

b) Scale factor $\frac{13}{4}$

Write the scale factor as a decimal.

____

Length of original photo: ______________________________
Length of enlargement: ________________ $=$ _____________

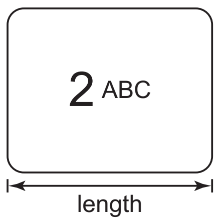
Width of original photo: ______________________________
Width of enlargement: ________________ $=$ _____________

The enlargement has dimensions _________________________.

Practice

1. Find the scale factor for each scale diagram.

a) The actual length of the cell phone button is 9 mm.



Measure the length of the button in the diagram.
Length $=$ _________ cm, or _________ mm

Scale factor $= \frac{\text{length on scale diagram}}{\text{length of button}} = \frac{\_\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_\_} =$ _______

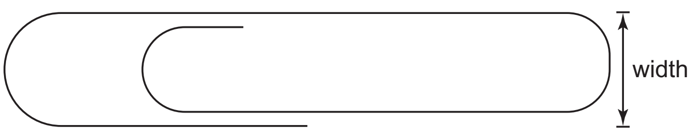
The scale factor is _______.

b) The actual width of the paperclip is 6 mm.

The width of the paperclip in the diagram is: Width $=$ _______ cm, or _______ mm

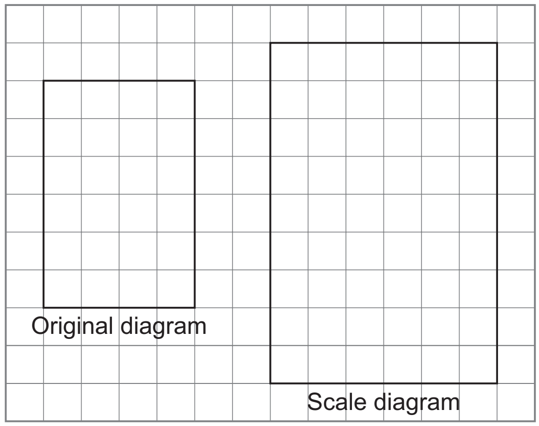
Scale factor $= \frac{\text{width on scale diagram}}{\text{width of paperclip}}$

$= \frac{\_\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_\_} =$ _______



The scale factor is _______.

2. Find the scale factor for this scale diagram.



Original length: ________
Length on scale diagram: ________

Scale factor $= \dfrac{\text{length on scale diagram}}{\text{length on original diagram}}$

$= \dfrac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$

$=$ ______

The scale factor is ______.

3. Enlargements of a photo are to be placed in different catalogues.
The original photo has side length 4 cm.
Find the side length for each enlargement of this photo.

a) Enlargement with scale factor 2.5

Side length of original photo: ________
Side length of enlargement: $2.5 \times$ ________ $=$ ________

The enlargement has side length ________.

b) Enlargement with scale factor $\frac{7}{4}$

Write the scale factor as a decimal:

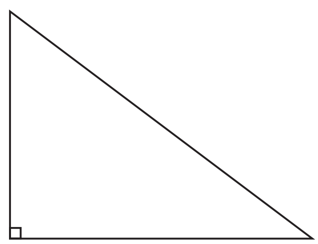
____

____

Side length of original photo: ________
Side length of enlargement: ______________ $=$ ________

The enlargement has side length ________.

4. Suppose you draw a scale diagram of this triangle.
You use a scale factor of 2.75.
What are the side lengths of the enlargement?



Side lengths of original triangle: ____________________
Scale factor: ________

Side lengths of enlargement:

____

____

____

____

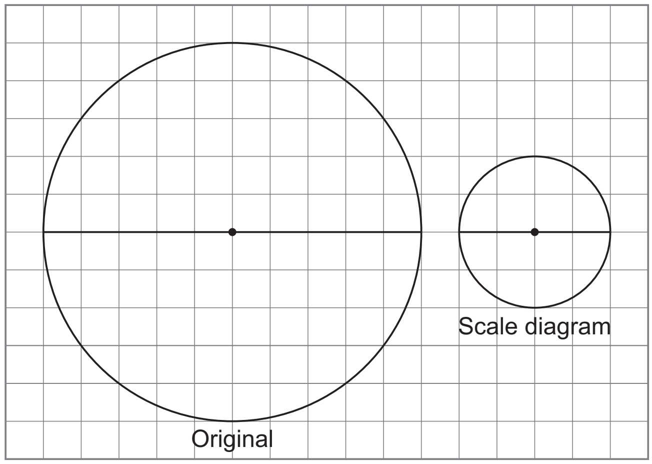
7.2 Scale Diagrams and Reductions

FOCUS **Draw and interpret scale diagrams that represent reductions.**

A scale diagram can be smaller than the original diagram.
This type of scale diagram is called a **reduction.**
A reduction has a scale factor that is less than 1.

Example 1 Using Matching Lengths to Determine the Scale Factor

Find the scale factor for this reduction.



Solution

Measure the diameter of the original circle. The diameter is 5 cm.
Measure the diameter of the scale diagram. The diameter is 2 cm.

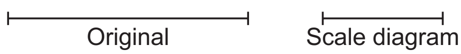
The scale factor is: $\frac{\text{diameter on scale diagram}}{\text{diameter on original diagram}} = \frac{2\text{ cm}}{5\text{ cm}} = \frac{2}{5}$

The scale factor is $\frac{2}{5}$. The scale factor is less than 1.

Check

1. Find the scale factor for each reduction.

a) Measure the length of the original line segment.
Length = ______ cm



Measure the length of the line segment in the scale diagram.
Length = ______ cm

Scale factor = $\frac{\text{length on scale diagram}}{\text{length on original diagram}}$

$= \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_} = \_\_\_\_\_\_$

The scale factor is _______.

b) Length of original rectangle: __________

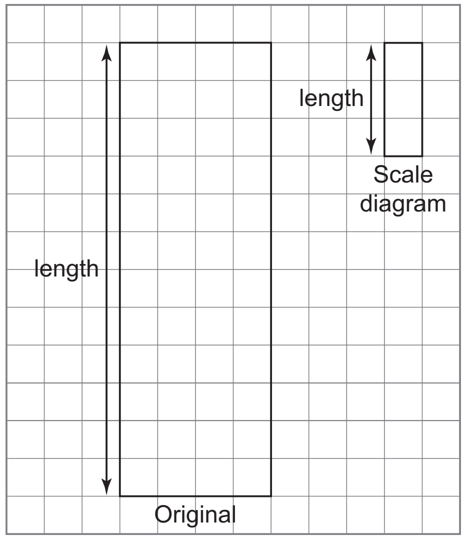
Length of rectangle in scale diagram: __________

$$\text{Scale factor} = \frac{\text{length on scale diagram}}{\text{length on original diagram}}$$

$$= \frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_}$$

$$= \_\_\_\_\_\_\_\_$$

The scale factor is _______.



Example 2 Using a Scale Factor to Determine Dimensions

The top view of a rectangular patio table has length 165 cm and width 105 cm.

A reduction is to be drawn with scale factor $\frac{1}{5}$.

Find the dimensions of the reduction.

Solution

Write the scale factor as a decimal.

$\frac{1}{5} = 1 \div 5 = 0.2$

Length of original table: 165 cm

Length of reduction: 0.2×165 cm $= 33$ cm

Width of original table: 105 cm

Width of reduction: 0.2×105 cm $= 21$ cm

The reduction has dimensions 33 cm by 21 cm.

Check

1. A window has dimensions 104 cm by 89 cm.

A reduction is to be drawn with scale factor $\frac{1}{20}$.

Find the dimensions of the reduction.

Write the scale factor as a decimal. $\frac{1}{20} =$ ________

Length of original window: _________

Length of reduction: _________________ = _________

Width of original window: _________

Width of reduction: _________________ = _________

The reduction has dimensions ______________________

2. The top view of a rectangular swimming pool has dimensions 10 m by 5 m.
A reduction is to be drawn with scale factor $\frac{1}{50}$.
Find the dimensions of the reduction.

Write the scale factor as a decimal.

____

Length of pool: _________

Length of reduction: ____________________

Convert this length to centimetres:

1 m = 100 cm

So, __________________________

____

Width of pool: _________

Width of reduction: ____________________

Convert this width to centimetres:

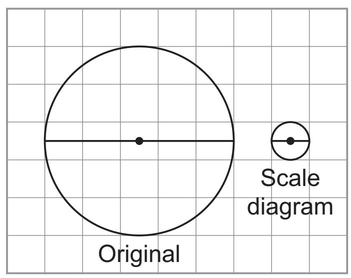
____

____

The reduction has dimensions __________________.

Practice

1. Find the scale factor for each reduction.



a) Diameter of original circle: _______ cm
Diameter of reduction: _______ cm

Scale factor $= \frac{\text{diameter on scale diagram}}{\text{diameter on original diagram}}$

$= \frac{\_\_\_\_}{\_\_\_\_}$

$=$ _______

The scale factor is _______.

b) Length of original line segment: _______
Length of reduction: _______

Scale factor $= \frac{\text{length on scale diagram}}{\text{length on original diagram}}$

$= \frac{\_\_\_\_}{\_\_\_\_}$

$=$ _______

The scale factor is _______.

2. A line segment has length 36 cm.
A reduction is to be drawn with scale factor $\frac{3}{20}$.
Draw a line segment with the new length.

Write the scale factor as a decimal.

____

Original length: ________

Length of reduction: ______________ = _________

Draw the line segment:

3. A reduction of each object is to be drawn with the given scale factor.
Find the matching length in centimetres on the reduction.

a) A water ski has length 170 cm.
The scale factor is 0.04.
Length of water ski: ________
Length of reduction: ______________ = _________

b) A canoe has length 4 m.
The scale factor is $\frac{3}{50}$.
Write the scale factor as a decimal.

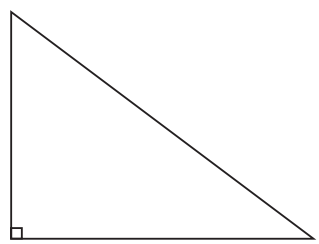
____

Length of canoe: ________

Length of reduction: ______________ = _________

Convert this length to centimetres: ______________________
____

4. Suppose you draw a scale diagram of this triangle.
You use a scale factor of $\frac{1}{4}$.
What are the side lengths of the reduction?



Side lengths of original triangle: ______________
Write the scale factor as a decimal.

____

Side lengths of reduction:

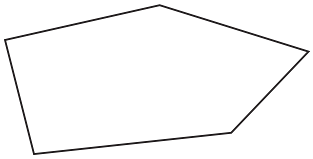
____
____
____
____
____

7.3 Skill Builder

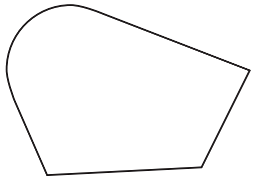
Polygons

A **polygon** is a closed shape with straight sides.
Exactly 2 sides meet at a vertex.

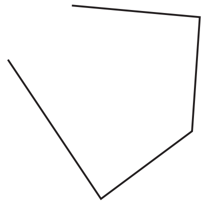
This shape is a polygon.



These shapes are **non-polygons.**



This shape has a curved side.

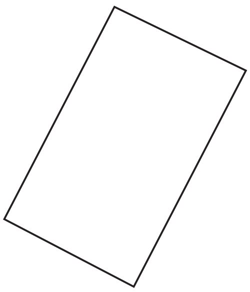


This shape is not closed.

Check

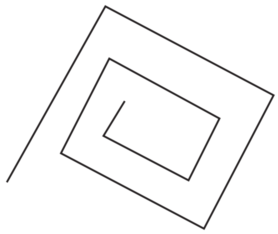
1. Is each shape a polygon or a non-polygon?

a)



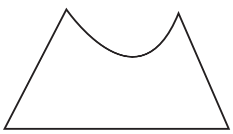
____

b)



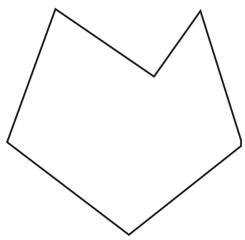
____

c)



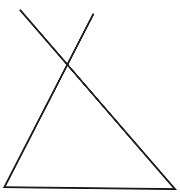
____

d)



____

e)



____

f)

____

7.3 Similar Polygons

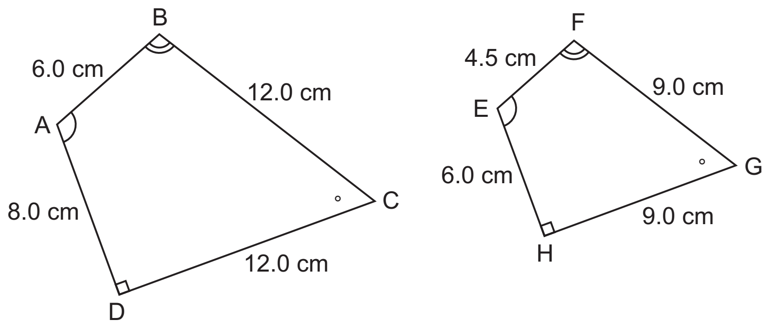
FOCUS **Recognize similar polygons, then use their properties to solve problems.**

When one polygon is an enlargement or reduction of another polygon,
we say the polygons are **similar.**
Similar polygons have the same shape, but not necessarily the same size.

When two polygons are similar:

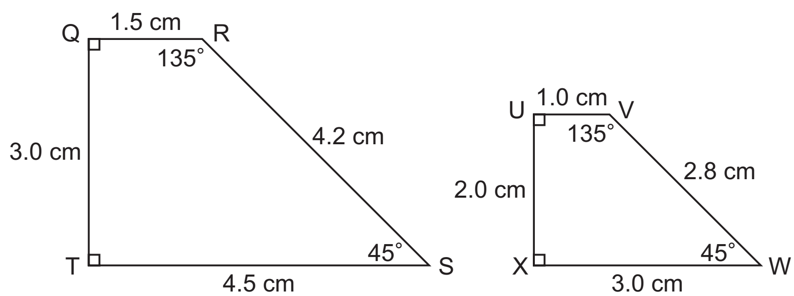
- matching angles are equal **AND**
- matching sides are proportional

When all pairs of matching sides have the same scale factor, we say matching sides are ***proportional.***



Example 1 Identifying Similar Polygons

Are these quadrilaterals similar?
Explain.



Solution

Check matching angles: $\angle Q = \angle U = 90°$ $\quad \angle R = \angle V = 135°$

$\angle S = \angle W = 45°$ $\quad \angle T = \angle X = 90°$

All matching angles are equal.
So, the first condition for similar polygons is met.

Check matching sides.
The matching sides are: QR and UV, RS and VW, ST and WX, and TQ and XU.
Find the scale factors.

$\frac{\text{length of QR}}{\text{length of UV}} = \frac{1.5\text{ cm}}{1.0\text{ cm}}$
$= 1.5$

$\frac{\text{length of RS}}{\text{length of VW}} = \frac{4.2\text{ cm}}{2.8\text{ cm}}$
$= 1.5$

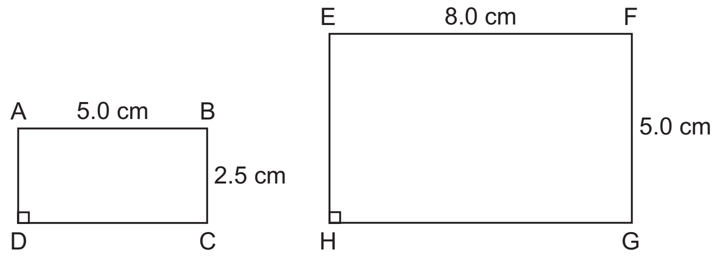
$\frac{\text{length of ST}}{\text{length of WX}} = \frac{4.5\text{ cm}}{3.0\text{ cm}}$
$= 1.5$

$\frac{\text{length of TQ}}{\text{length of XU}} = \frac{3.0\text{ cm}}{2.0\text{ cm}}$
$= 1.5$

All scale factors are equal, so matching sides are proportional.
Since matching angles are equal and matching sides are proportional,
the quadrilaterals are similar.

Check

1. Are these rectangles similar?



Check matching angles.
The measure of each angle in
a rectangle is _______.
So, matching angles are _______.

Check matching sides.
The matching sides are: ____ and ____, and ____ and ____.
Find the scale factors.

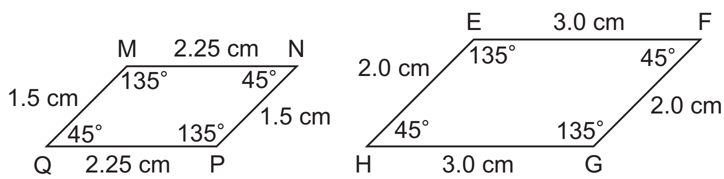
Since opposite sides of a rectangle are equal, check only one pair of matching lengths and one pair of matching widths.

$\frac{\text{length of \_\_\_\_\_}}{\text{length of \_\_\_\_\_}} = \frac{\text{\_\_\_\_\_\_\_\_\_\_}}{\text{\_\_\_\_\_\_\_\_\_\_}}$ $\qquad$ $\frac{\text{length of \_\_\_\_\_}}{\text{length of \_\_\_\_\_}} = \frac{\text{\_\_\_\_\_\_\_\_\_\_}}{\text{\_\_\_\_\_\_\_\_\_\_}}$

$= \text{\_\_\_\_\_\_\_\_\_\_}$ $\qquad$ $= \text{\_\_\_\_\_\_\_}$

The scale factors ________ equal.
So, the sides ________ proportional.
The rectangles ________ similar.

2. Are these parallelograms similar?



Check matching angles. $\angle M$ = _____ = _____
$\angle N$ = _____ = _____
____
____

All matching angles ________ equal.

Check matching sides.
The matching sides are: _____ and _____, and _____ and _____.
Find the scale factors.

Since opposite sides of a parallelogram are equal, check only two pairs of matching sides.

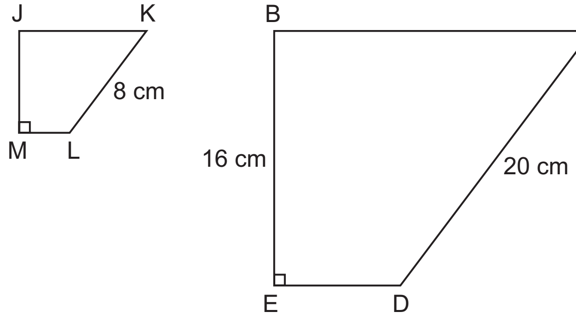
$\frac{\text{length of \_\_\_\_\_}}{\text{length of \_\_\_\_\_}} = \frac{\text{\_\_\_\_\_\_\_\_\_\_}}{\text{\_\_\_\_\_\_\_\_\_\_}}$ $\qquad$ $\frac{\text{length of \_\_\_\_\_}}{\text{length of \_\_\_\_\_}} = \frac{\text{\_\_\_\_\_\_\_\_\_\_}}{\text{\_\_\_\_\_\_\_\_\_\_}}$

$= \text{\_\_\_\_\_\_\_}$ $\qquad$ $= \text{\_\_\_\_\_\_\_}$

The scale factors ________ equal.
So, the sides ________ proportional.
The parallelograms ________ similar.

Example 2 Determining Lengths in Similar Polygons

These two quadrilaterals are similar.
Find the length of JM.



Solution

Quadrilateral JKLM is a reduction of quadrilateral BCDE.
To find the scale factor of the reduction, choose a pair of matching sides whose lengths are both known:
CD = 20 cm and KL = 8 cm

Consider the polygon with the unknown length as a reduction or enlargement of the other polygon.

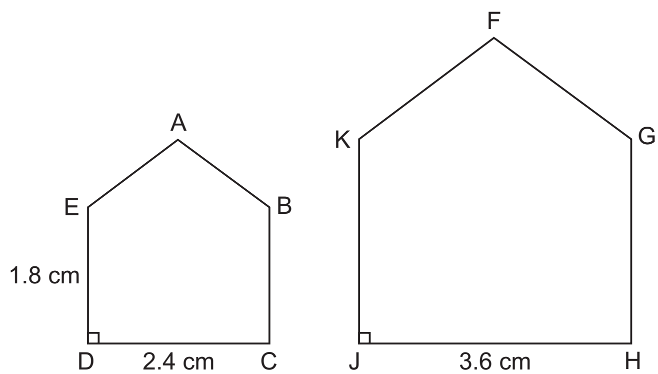
$$\text{Scale factor} = \frac{\text{length on reduction}}{\text{length on original}}$$
$$= \frac{8 \text{ cm}}{20 \text{ cm}}$$
$$= 0.4$$

The scale factor is 0.4.
Use the scale factor to find the length of JM.
JM and BE are matching sides.
Length of BE: 16 cm
Scale factor: 0.4
Length of JM: 0.4×16 cm = 6.4 cm

So, JM has length 6.4 cm.

Check

1. These two polygons are similar.
Find the length of JK.



Polygon FGHJK is an enlargement of polygon ABCDE.
To find the scale factor, choose a pair of matching sides whose lengths are both known:

____

Scale factor $= \dfrac{\text{length on enlargement}}{\text{length on original}}$

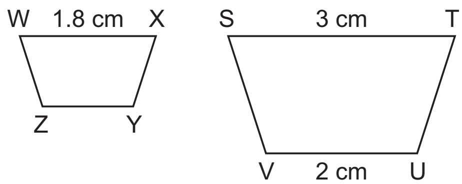
$= \dfrac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_}$

$= \_\_\_\_\_$

The scale factor is _____.
Use the scale factor to find the length of JK.
JK and DE are matching sides.
Length of DE: ________
Scale factor: _____
Length of JK: _________________________

So, JK has length ________.

2. These two polygons are similar.
Find the length of YZ.



Polygon WXYZ is a ___________ of polygon STUV.
To find the scale factor, choose a pair of matching sides whose lengths are both known:

____

Scale factor $= \dfrac{\text{length on }\_\_\_\_\_\_\_\_}{\text{length on original}}$

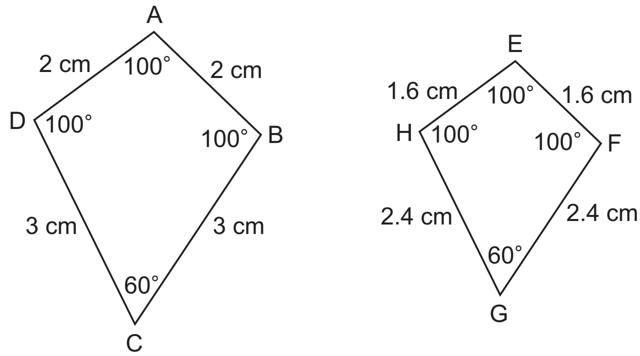
$= \dfrac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_}$

$= \_\_\_\_\_$

The scale factor is _____.
Use the scale factor to find the length of YZ.
UV and YZ are matching sides.
Length of UV: ________
Scale factor: _____
Length of UV: _________________________
So, UV has length ________.

Practice

1. Are these quadrilaterals similar?



Check matching angles. ∠A = _____ = _____

∠B = _____ = _____

____

____

All matching angles _______ equal.

Check matching sides.

The matching sides are: AB and ____, and BC and ____.

Find the scale factors.

Since adjacent sides of the kites are equal, check only two pairs of matching sides.

$\dfrac{\text{length of \_\_\_\_\_}}{\text{length of \_\_\_\_\_}} = \dfrac{\_\_\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_\_\_}$ = _______

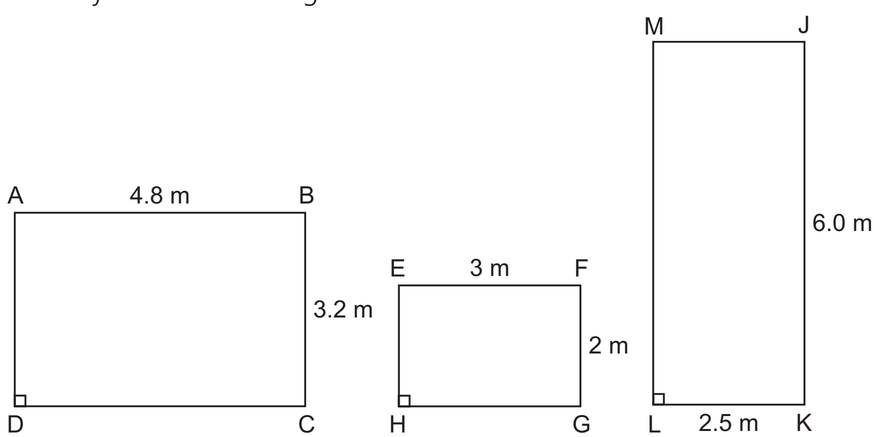
$\dfrac{\text{length of \_\_\_\_\_}}{\text{length of \_\_\_\_\_}} = \dfrac{\_\_\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_\_\_}$ = _______

The scale factors ________ equal.

So, the sides ________ proportional.

The quadrilaterals ________ similar.

2. Are any of these rectangles similar?



The measure of each angle in a rectangle is ____.

So, for any two rectangles, matching angles are _______.

Check matching lengths and widths in pairs of rectangles.

For rectangles ABCD and EFGH, the scale factors are:

$\dfrac{\text{length of \_\_\_\_\_}}{\text{length of \_\_\_\_\_}} = \dfrac{\_\_\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_\_\_}$ = _______

$\dfrac{\text{length of \_\_\_\_\_}}{\text{length of \_\_\_\_\_}} = \dfrac{\_\_\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_\_\_}$ = _______

The scale factors _______ equal.

So, the sides _______ proportional.

The rectangles _______ similar.

For rectangles ABCD and JKLM, the scale factors are:

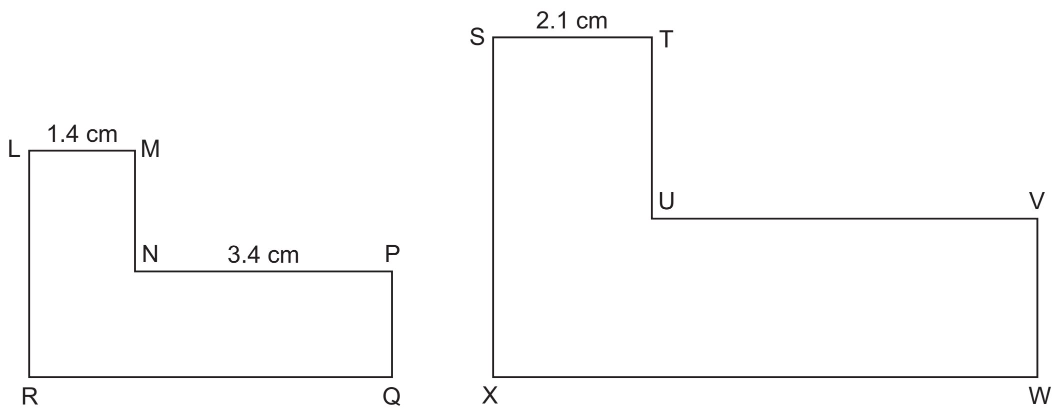
$$\frac{\text{length of }\underline{\hspace{1cm}}}{\text{length of }\underline{\hspace{1cm}}} = \frac{\underline{\hspace{2.5cm}}}{\underline{\hspace{2.5cm}}} \qquad \frac{\text{length of }\underline{\hspace{1cm}}}{\text{length of }\underline{\hspace{1cm}}} = \frac{\underline{\hspace{2.5cm}}}{\underline{\hspace{2.5cm}}}$$

$$= \underline{\hspace{1.8cm}} \qquad\qquad\qquad = \underline{\hspace{1.8cm}}$$

The scale factors ____________ equal.
So, the sides ____________ proportional.
The rectangles ____________ similar.

Is rectangle EFGH similar to rectangle JKLM?
Use what we know to find out.
We know that rectangle ABCD ___________________ to rectangle EFGH.
We know that rectangle ABCD ___________________ to rectangle JKLM.
So, we know rectangle EFGH ___________________ to rectangle JKLM.

3. These two polygons are similar.
Find the length of UV.



Polygon STUVWX is an enlargement of polygon LMNPQR.
To find the scale factor, choose a pair of matching sides whose lengths are both known:

____

$$\text{Scale factor} = \frac{\text{length on enlargement}}{\text{length on original}}$$

$$= \frac{\underline{\hspace{2.5cm}}}{\underline{\hspace{2.5cm}}}$$

$$= \underline{\hspace{1.8cm}}$$

The scale factor is __________.
Use the scale factor to find the length of UV.
UV and NP are matching sides.
Length of NP: __________
Scale factor: __________
Length of UV: _________________________
So, UV has length __________.

7.4 Skill Builder

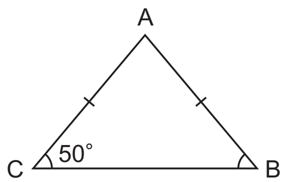
Sum of the Angles in a Triangle

In any triangle, the sum of the angle measures is 180°.

So, to find an unknown angle measure:
- start with 180°
- subtract the known measures

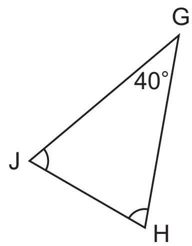
An isosceles triangle has 2 equal sides and 2 equal angles.

To find the measure of the third angle, subtract the measure of the equal angles twice.



$\angle A = 180° - 50° - 50°$
$= 80°$

To find the measure of each equal angle, subtract the known angle from 180°, then divide by 2.

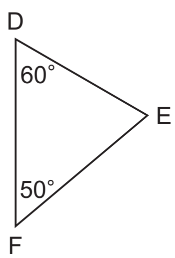


Sum of equal angles is: $180° - 40° = 140°$
Measure of each equal angle: $140° \div 2 = 70°$

Check

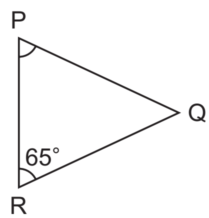
1. Find the measure of the third angle.

a)



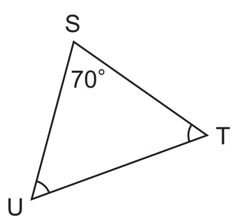
∠E = 180° − _____ − _____
= _____

b)



∠Q = ____________________
= _____

2. Find the measure of each equal angle.



Sum of equal angles is:
180° − _____ = _____
Measure of each equal angle:
_____ ÷ 2 = _____

7.4 Similar Triangles

FOCUS **Use the properties of similar triangles to solve problems.**

A triangle is a special polygon.
When two triangles are similar:

- matching angles are equal **OR**
- matching sides are proportional

The order in which similar triangles are named gives a lot of information.

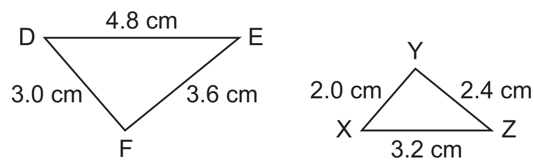
Suppose $\triangle ABC \sim \triangle DEF$.

The symbol ~ means "is similar to."

Then, $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$
Similarly, AB matches DE, BC matches EF, and AC matches DF.

Example 1 Identifying Similar Triangles

Name the similar triangles.



Solution

Angle measures are not given.
So, find out if matching sides are proportional.
In $\triangle DEF$, order the sides from shortest to longest: FD, EF, DE
In $\triangle XYZ$, order the sides from shortest to longest: XY, YZ, ZX
Find the scale factors of matching sides.

$$\frac{\text{length of FD}}{\text{length of XY}} = \frac{3.0\text{ cm}}{2.0\text{ cm}} = 1.5 \qquad \frac{\text{length of EF}}{\text{length of YZ}} = \frac{3.6\text{ cm}}{2.4\text{ cm}} = 1.5 \qquad \frac{\text{length of DE}}{\text{length of ZX}} = \frac{4.8\text{ cm}}{3.2\text{ cm}} = 1.5$$

Since all scale factors are the same, the triangles are similar.

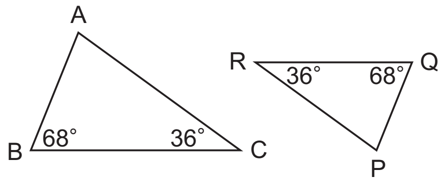
The longest and shortest sides meet at vertices: D and X
The two longer sides meet at vertices: E and Z
The two shorter sides meet at vertices: F and Y
So, $\triangle DEF \sim \triangle XZY$

Read the letters down the columns.

Check

1. In each diagram, name two similar triangles.

a) Two angles in each triangle are given.
The measure of the third angle
in each triangle is:
180° − ________________



List matching angles:
∠A = _____ = _____
∠B = _____ = _____
∠C = _____ = _____
Matching angles _______ equal.
So, the triangles _______ similar.

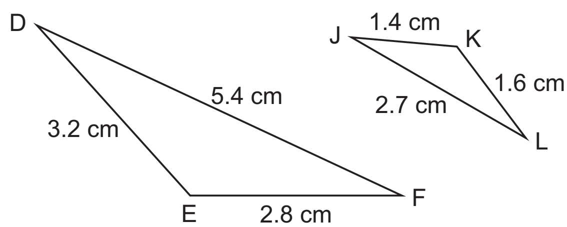
To name the triangles, order the letters so matching angles correspond.
△ABC ~ △_______

b) Find out if matching sides are proportional.
In △DEF, order the sides from shortest to longest:

____

In △JKL, order the sides from shortest to longest:

____



Find the scale factors of matching sides.

$\frac{\text{length of ____}}{\text{length of ____}} = \frac{\text{__________}}{\text{__________}} = \text{____}$

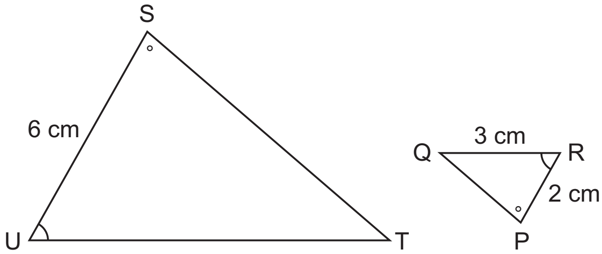
$\frac{\text{length of ____}}{\text{length of ____}} = \frac{\text{__________}}{\text{__________}} = \text{____}$

$\frac{\text{length of ____}}{\text{length of ____}} = \frac{\text{__________}}{\text{__________}} = \text{____}$

All scale factors are ___________. So, the triangles _______________.
The two longer sides meet at vertices: ____ and ____
The two shorter sides meet at vertices: ____ and ____
The longest and shortest sides meet at vertices: ____ and ____
So, △DEF ~ △______

Example 2 Using Similar Triangles to Determine a Length

These two triangles are similar.
Find the length of TU.



Solution

List matching angles:
$\angle S = \angle P$ $\angle T = \angle Q$ $\angle U = \angle R$
So, $\triangle STU \sim \triangle PQR$

$\triangle STU$ is an enlargement of $\triangle PQR$.

Choose a pair of matching sides
whose lengths are both known:
SU = 6 cm and PR = 2 cm

Consider the triangle with the unknown length as a reduction or enlargement of the other triangle.

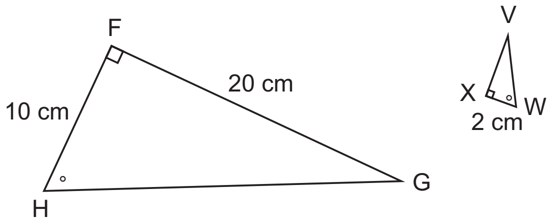
$$\text{Scale factor} = \frac{\text{length on enlargement}}{\text{length on original}}$$
$$= \frac{6 \text{ cm}}{2 \text{ cm}}$$
$$= 3$$

The scale factor is 3.
Use the scale factor to find the length of TU.
TU and QR are matching sides.
Length of QR: 3 cm
Scale factor: 3
Length of TU: 3×3 cm = 9 cm

So, TU has length 9 cm.

Check

1. These two triangles are similar.
Find the length of XV.



List matching angles:

∠F = _____ ∠G = _____ ∠H = _____

So, △FGH ~ △_______

_________ is a reduction of _________.

Choose a pair of matching sides whose lengths are both known:

____

Scale factor = $\frac{\text{length on reduction}}{\text{length on original}}$

= $\frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_}$

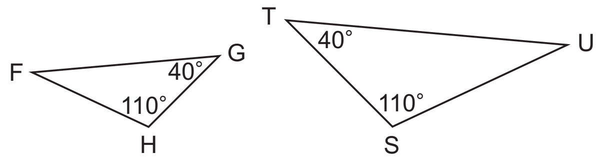
= _____

The scale factor is _____.
Use the scale factor to find the length of XV.
XV and FG are matching sides.
Length of FG: ________
Scale factor: ______
Length of XV: _________________________

So, XV has length ______.

Practice

1. In each diagram, name two similar triangles.



a) Two angles in each triangle are given.
The measure of the third angle
in each triangle is: 180° − ______________

List matching angles:

∠F = _____ = _____ ∠G = _____ = _____ ∠H = _____ = _____

Matching angles _______ equal, so, the triangles _______ similar.

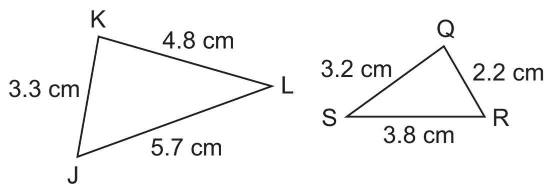
To name the triangles, order the letters so matching angles correspond. △FGH ~ △______

b) Find out if matching sides are proportional.

In △JKL, order the sides from shortest to longest: ____________

In △QRS, order the sides from shortest to longest: ____________

Find the scale factors of matching sides.



length of ____ / length of ____ = ________ / ________ = ____

length of ____ / length of ____ = ________ / ________ = ____

length of ____ / length of ____ = ________ / ________ = ____

All scale factors are ___________. So, the triangles ___________.

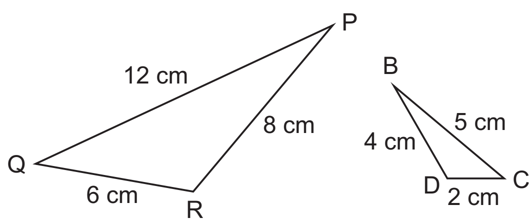
The longest and shortest sides meet at vertices: ____ and ____

The two shorter sides meet at vertices: ____ and ____

The two longer sides meet at vertices: ____ and ____

So, △JKL ~ △______

2. Are these two triangles similar?



In △PQR, order the sides from shortest to longest:

____

In △BCD, order the sides from shortest to longest:

____

Find the scale factors of matching sides.

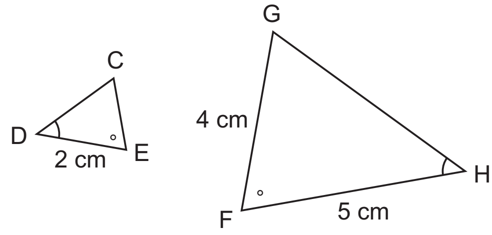
length of _____ / length of _____ = ________ / ________ = ____

length of _____ / length of _____ = ________ / ________ = ____

length of _____ / length of _____ = ________ / ________ = ____

All scale factors are ____________. So, the triangles _______________.

3. These two triangles are similar.
Find the length of EC.



List matching angles:

$\angle C$ = ______ $\angle D$ = ______ $\angle E$ = ______

So, $\triangle CDE \sim \triangle$_______

________ is a reduction of ________.

Choose a pair of matching sides whose lengths are both known:

____

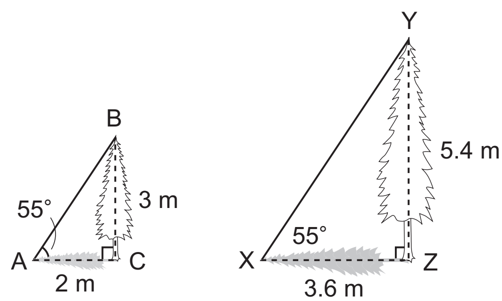
Scale factor $= \frac{\text{length on reduction}}{\text{length on original}}$

$= \frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_}$

$=$ _____

The scale factor is _____.
Use the scale factor to find the length of EC.
EC and _____ are matching sides.
Length of _____: _______
Scale factor: _____
Length of EC: __________________________
So, EC has length ________.

4. At a certain time of day, two trees cast shadows.
Find the height of the taller tree.



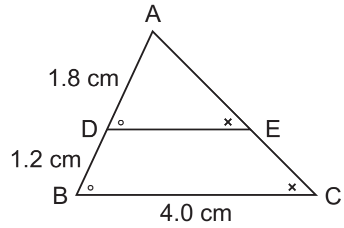
Matching angles are _________.
So, $\triangle ABC \sim \triangle$_______
$\triangle XYZ$ is an ________________ of $\triangle ABC$.
Use sides ______________________________
to find the scale factor.

$\frac{\text{length on enlargement}}{\text{length on original}} = \frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_}$

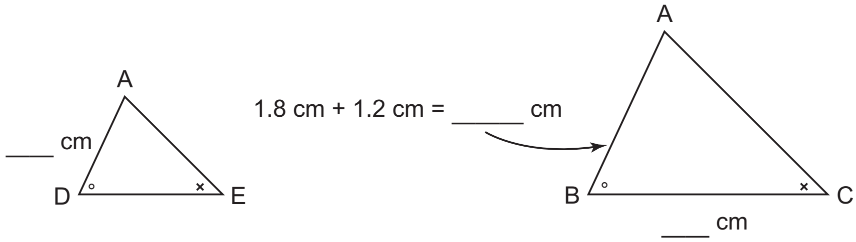
$=$ _____

The scale factor is 1.8.
Use the scale factor to find the height of the taller tree, YZ.
BC and YZ are matching sides.
Length of BC: ______ Scale factor: _____
Length of YZ: _____________________
So, the height of the taller tree is _____________.

5. The two triangles in this diagram are similar.
Find the length of DE.



To better see the individual triangles, we draw the triangles separately.



∠A = ______ ∠B = ______ ∠C = ______

So, △ABC ~ △______

_________ is a reduction of _________.

Choose a pair of matching sides whose lengths are both known:

____

$$\text{Scale factor} = \frac{\text{length on reduction}}{\text{length on original}}$$

$$= \frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_}$$

$$= \_\_\_\_\_$$

The scale factor is _____.

Use the scale factor to find the length of DE.

_____ and _____ are matching sides.

Length of _____: __________

Scale factor: _____

Length of DE: _________________________

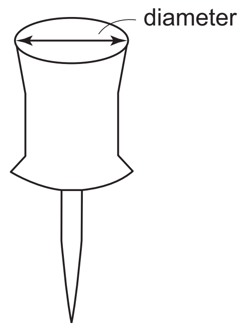
So, DE has length _________.

CHECKPOINT

Can you ...

- Find the scale factor for a scale diagram?
- Use a scale factor to determine a length?
- Identify similar polygons and triangles?
- Use similar polygons and triangles to determine a length?

7.1 **1.** Find the scale factor for this scale diagram.
The actual diameter of the head of the pushpin is 6 mm.



Measure the diameter of the pushpin in the diagram.
Length = ________ cm, or ________ mm

Scale factor = $\dfrac{\text{length on scale diagram}}{\text{length of pushpin}}$

= $\dfrac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$

= _____

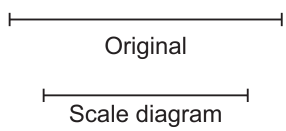
The scale factor is _____.

2. A baby picture is to be enlarged.
The dimensions of the photo are 5 cm by 7 cm.
Find the dimensions of the enlargement with a scale factor of 3.2.
Length of original photo: ______
Length of enlargement: 3.2 × ______ = __________

Width of original photo: ______
Width of enlargement: ______ × ______ = _______

The enlargement has dimensions __________________________.

7.2 **3.** Find the scale factor for this reduction.
Length of original line segment: ______ cm
Length of reduction: ______ cm



Scale factor = $\dfrac{\text{length on reduction}}{\text{length on original diagram}}$

= $\dfrac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$

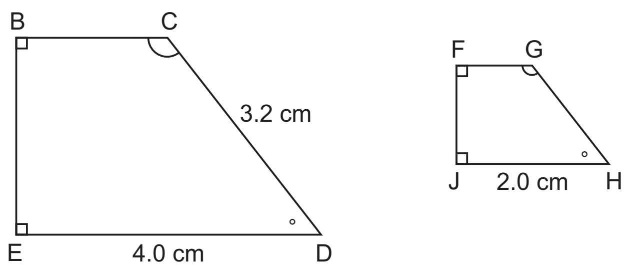
= _____

The scale factor is _____.

4. A reduction of a lacrosse stick is to be drawn with a scale factor of $\frac{7}{50}$.
The lacrosse stick has length 100 cm.
Find the length of the reduction.

Write the scale factor as a decimal.
$\frac{7}{50} =$ ______
Length of lacrosse stick: ________
Length of reduction: ________ $\times$ ________ $=$ ________
The reduction has length ________.

7.3 **5.** These two quadrilaterals are similar.
Find the length of GH.



Quadrilateral FGHJ is a _____________ of quadrilateral BCDE.
To find the scale factor, choose a pair of matching sides whose lengths are both known:

____

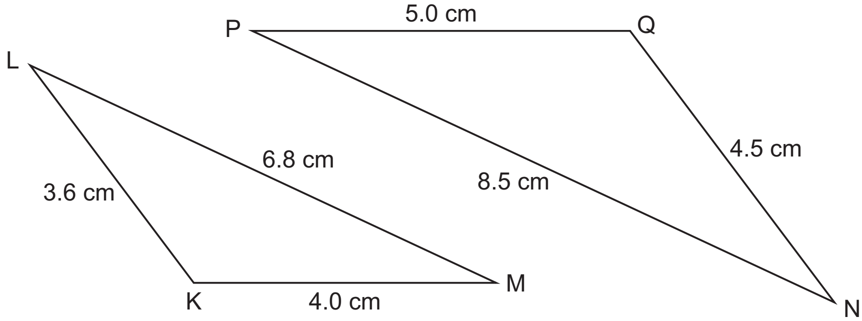
Scale factor $= \frac{\text{length on \_\_\_\_\_\_\_\_\_\_}}{\text{length on original}}$

$= \frac{\_\_\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_\_\_}$

$=$ _____

The scale factor is _____.
Use the scale factor to find the length of GH.
GH and _____ are matching sides.
Length of _____: __________
Scale factor: _____
Length of GH: ________________________
So, GH has length _______.

7.4 **6.** Are these 2 triangles similar?



Find out if matching sides are proportional.

In △KLM, order the sides from shortest to longest: ____________________

In △NPQ, order the sides from shortest to longest: ____________________

Find the scale factors of matching sides.

$\frac{\text{length of \_\_\_\_}}{\text{length of \_\_\_\_}} = \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_} = \_\_\_\_$

$\frac{\text{length of \_\_\_\_}}{\text{length of \_\_\_\_}} = \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_} = \_\_\_\_$

$\frac{\text{length of \_\_\_\_}}{\text{length of \_\_\_\_}} = \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_} = \_\_\_\_$

All scale factors are __________. So, the triangles __________.

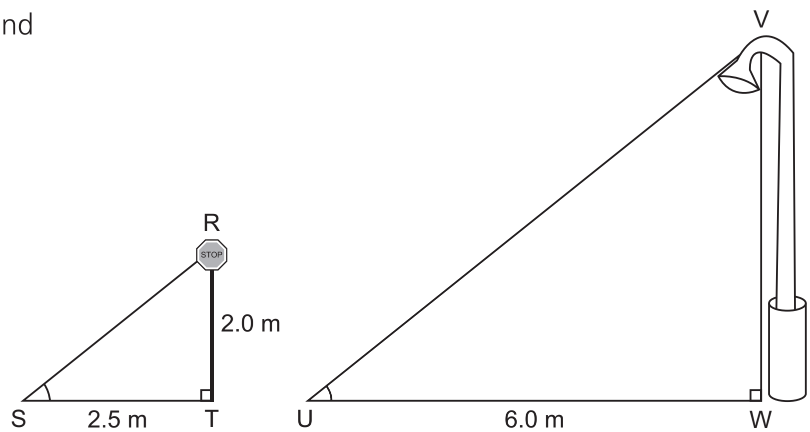
The two shorter sides meet at vertices: ____ and ____

The longest and shortest sides meet at vertices: ____ and ____

The two longer sides meet at vertices: ____ and ____

So, △KLM ~ △______

7. At a certain time of day, a street light and a stop sign cast shadows.
Find the height of the street light.



Matching angles are ________.

So, △RST ~ △________

△________ is an enlargement of △________.

Use sides ____________ and ______________ to find the scale factor.

$\frac{\text{length on enlargement}}{\text{length on original}} = \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$

$= \_\_\_\_$

The scale factor is ____.

Use the scale factor to find the height of the street light, VW.

VW and ____ are matching sides.

Length of ____ : ______ Scale factor: ____

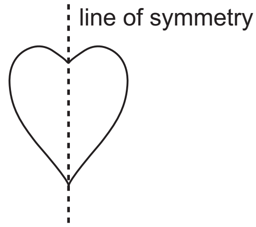
Length of VW: ____________________________

So, the height of the street light is ______.

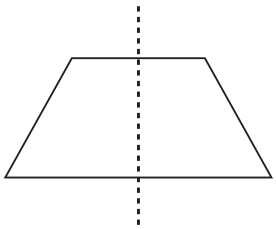
7.5 Skill Builder

Lines of Symmetry in Quadrilaterals

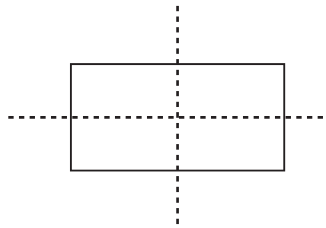
A **line of symmetry** divides a shape
into 2 matching, or **congruent** parts.
If we fold a shape along its line of symmetry,
the parts match exactly.



This trapezoid has 1 line of symmetry.



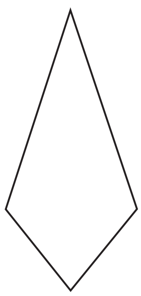
This rectangle has 2 lines of symmetry.



Check

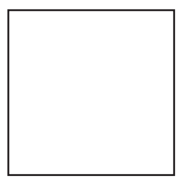
1. How many lines of symmetry does each shape have? Draw in the lines.

a)



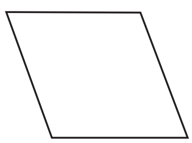
Number of lines of symmetry: ____

b)



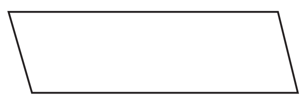
Number of lines of symmetry: ____

c)



Number of lines of symmetry: ____

d)



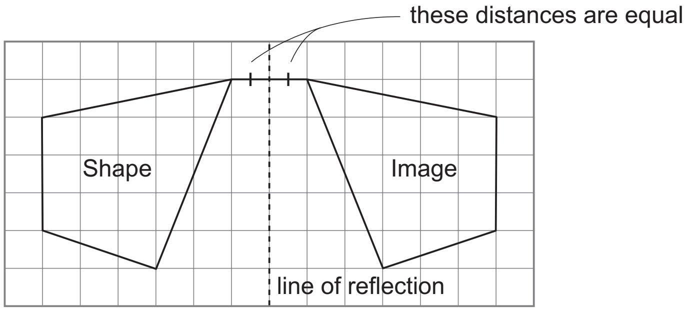
Number of lines of symmetry: ____

Reflections

When a shape is reflected in a mirror, we see a **reflection image.**

A point and its reflection image are the same distance from a **line of reflection.**

A shape and its reflection image face opposite ways.

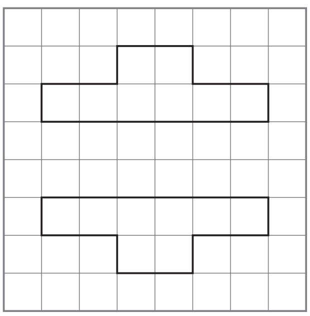


We can use a Mira to help us reflect a shape.

Check

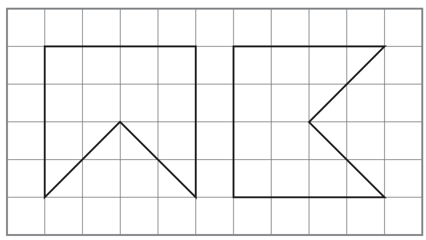
1. Do these pictures show reflections?
If your answer is Yes, draw the line of reflection.

a)



______

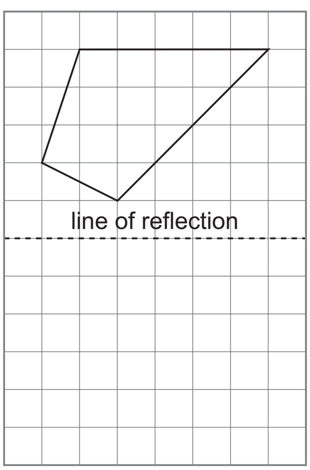
b)



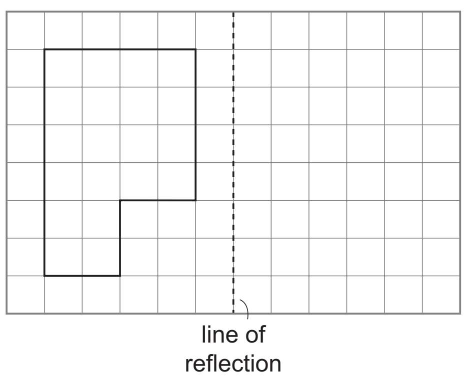
______

2. Draw each reflection image.

a)



b)



7.5 Reflections and Line Symmetry

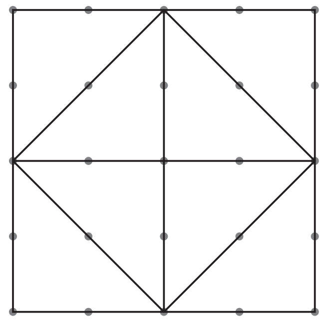
FOCUS **Draw and classify shapes with line symmetry.**

When congruent copies of a polygon are used to cover a flat surface with no overlaps or gaps, a **tessellation** is created. Some tessellations have line symmetry.

Congruent polygons match exactly but may have different orientations.

Example 1 Identifying Lines of Symmetry in Tessellations

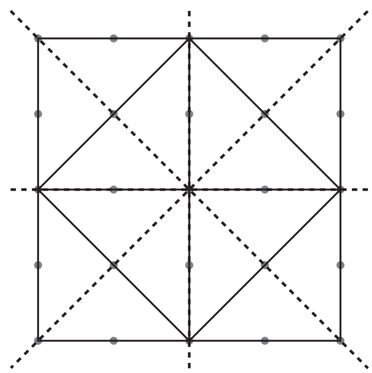
Identify the lines of symmetry in this tessellation.



Solution

A line of symmetry must pass through the centre of the design. Use a Mira to check for vertical, horizontal, and diagonal lines of symmetry.

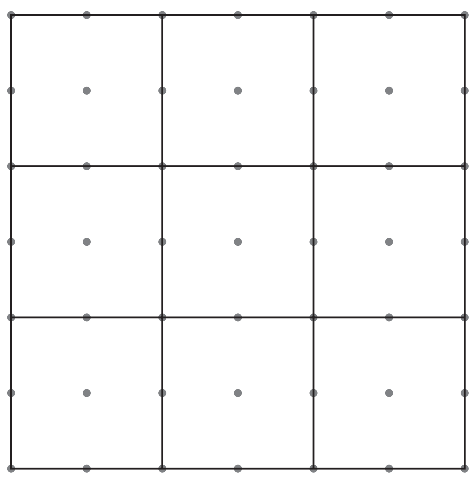
This tessellation has 4 lines of symmetry. The pattern on one side of each line is a mirror image of the pattern on the other side of the line.



Check

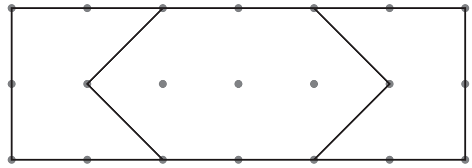
1. Draw the lines of symmetry in each tessellation.

a) Use a Mira.
Is there a vertical line of symmetry? ______
Is there a horizontal line of symmetry? ______
Are there any diagonal lines of symmetry? ______
Draw the lines of symmetry.



Remember that a line of symmetry must pass through the centre of the design.

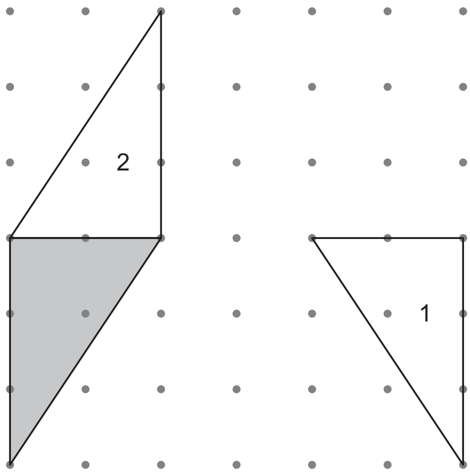
b) Is there a vertical line of symmetry? ______
Is there a horizontal line of symmetry? ______
Are there any diagonal lines of symmetry? ______
Draw the lines of symmetry.



Two shapes may be related by a line of reflection.

Example 2 Identifying Reflected Shapes

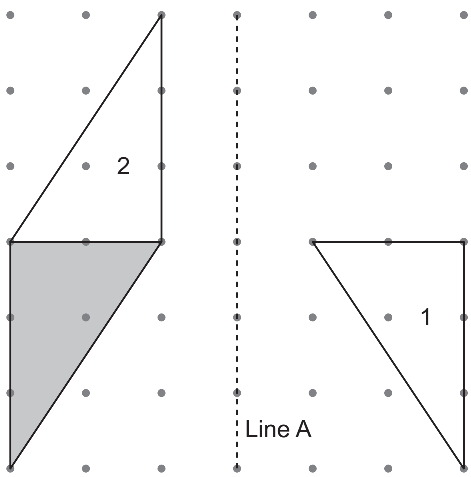
Which triangle is a reflection of the shaded triangle?
Draw the line of reflection.



Solution

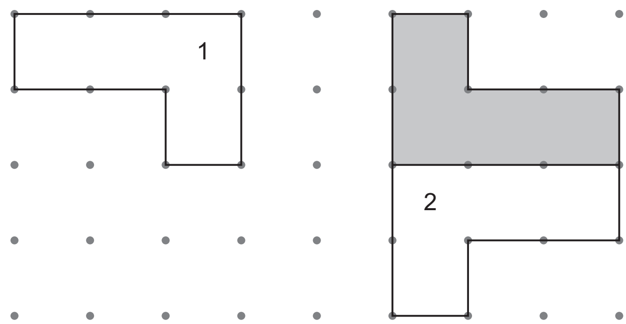
Use a Mira to check.
Triangle 1:
The triangle is to the right of the shaded triangle.
So, try a vertical line of reflection.
The triangle is the reflection image
of the shaded triangle in Line A.

Triangle 2:
The triangle is above the shaded triangle.
So, try a horizontal line of reflection.
The triangle is not a reflection image
of the shaded triangle.



Check

1. Which polygon is a reflection of the shaded polygon?
Draw the line of reflection.

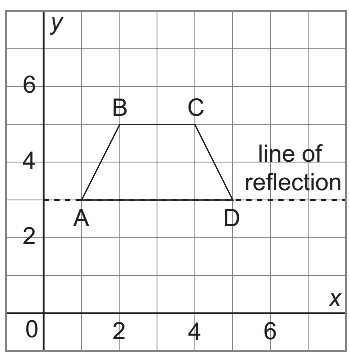


Use a Mira to check.
Polygon 1:
The polygon is to the __________ of the shaded polygon.
So, try a ____________ line of reflection.
The polygon __________ a reflection image
of the shaded polygon.
If the polygon is a reflection image, draw the line of reflection.

Polygon 2:
The polygon is __________ the shaded polygon.
So, try a ______________ line of reflection.
The polygon __________ a reflection image
of the shaded polygon.
If the polygon is a reflection image, draw the line of reflection.

Example 3 Completing a Shape Given its Line of Symmetry

Reflect quadrilateral ABCD in the line of reflection to make a larger shape.



Solution

A point and its image must be the same distance from the line of reflection.

Point A: on the line of reflection
Reflection image: Point A reflects onto itself.

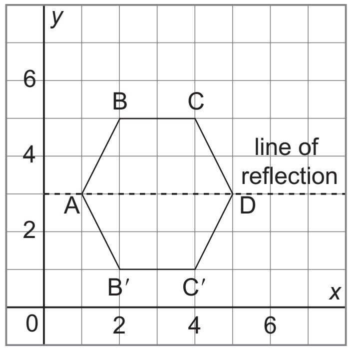
Point B: 2 squares above line of reflection
Reflection image: Point B′ is 2 squares below line of reflection.

Point C: 2 squares above line of reflection
Reflection image: Point C′ is 2 squares below line of reflection.

Point D: on the line of reflection
Reflection image: Point D reflects onto itself.

Plot the points. Join the points in order to complete the larger shape.

Point	Image
A(1, 3)	A(1, 3)
B(2, 5)	B′(2, 1)
C(4, 5)	C′(4, 1)
D(5, 3)	D(5, 3)



Point B′ is the image of point B. We say: "B prime"

Check

1. Reflect quadrilateral EFGH in the line of reflection to make a larger shape.

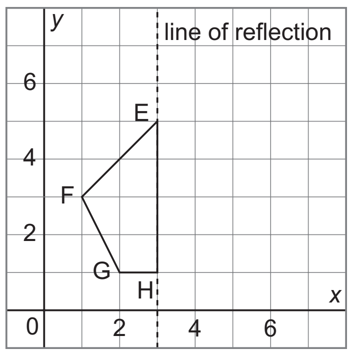
Point E: on the line of reflection
Reflection image: ______________________

Point F: 2 squares left of line of reflection
Reflection image: ______________________
____

Point G: ______________________
Reflection image: ______________________
____

Point H: ______________________
Reflection image: ______________________

Plot the points.
Join the points to complete the larger shape.

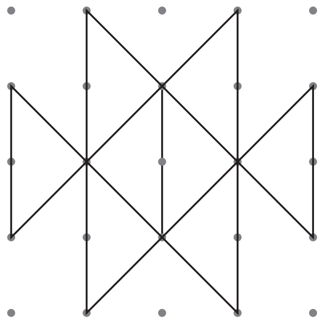


Point	Image
E(3, 5)	E(___, 5)
F(1, 3)	F′(___, 3)
G(2, 1)	G′(___, 1)
H(3, 1)	H(___, 1)

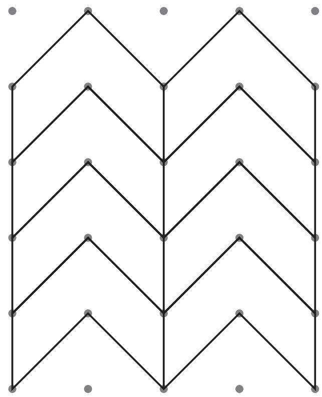
Practice

1. Draw the lines of symmetry in each tessellation.

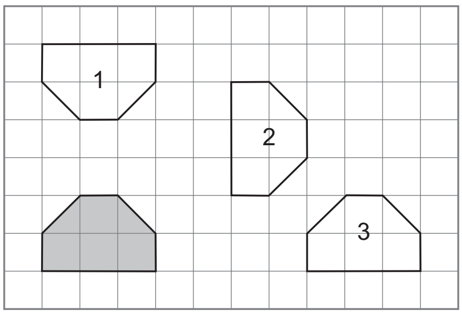
a)



b)



2. Which hexagons are reflections of the shaded hexagon?
Draw the line of reflection each time.



Hexagon 1:
The hexagon is ____________ the shaded hexagon.
So, try a ______________ line of reflection.
The hexagon ____________ a reflection image of the shaded hexagon.
If the polygon is a reflection image, draw the line of reflection, Line A.

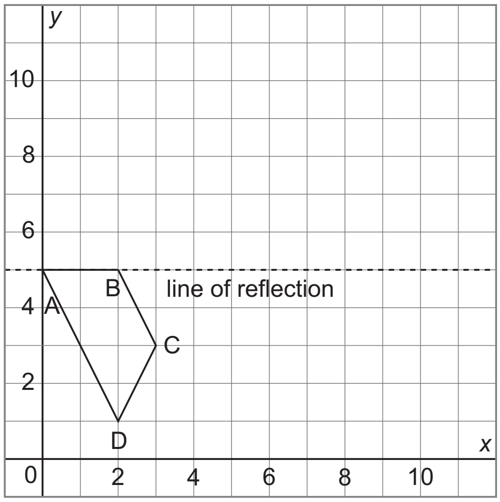
Hexagon 2:
The hexagon is ____________ and to the ____________ of the shaded polygon.
So, try a _______________ line of reflection.
The hexagon ____________ a reflection image of the shaded hexagon.
If the polygon is a reflection image, draw the line of reflection, Line B.

Hexagon 3:
The hexagon is to the ___________ of the shaded hexagon.
So, try a ___________ line of reflection.
The hexagon ____________ a reflection image of the shaded hexagon.
If the polygon is a reflection image, draw the line of reflection, Line C.

3. Reflect each shape in the line of reflection to make a larger shape.

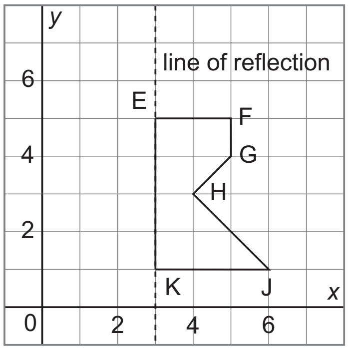
a)

Point	Image
A(0, 5)	A(___, ___)
B(2, 5)	B(___, ___)
C(3, 3)	C′(___, ___)
D(2, 1)	D′(___, ___)



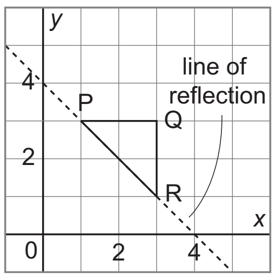
b)

Point	Image
________	________
________	________
________	________
________	________
________	________
________	________



c)

Point	Image
________	________
________	________
________	________



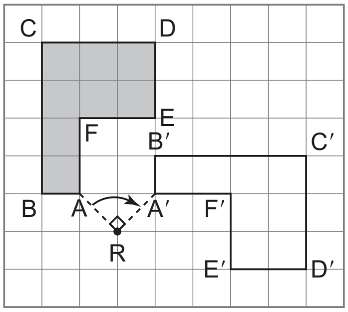
7.6 Skill Builder

Rotations

A **rotation** may be clockwise or counterclockwise.
Some common rotations are 90°, 180°, and 270°.

A complete turn measures 360°.

This shape was rotated 90° clockwise about point R.

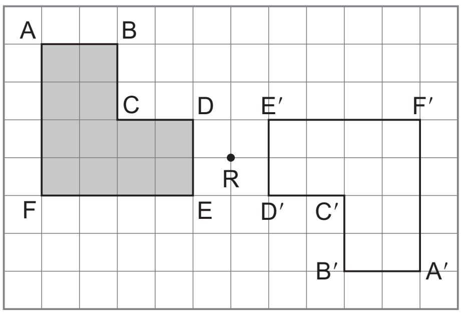


∠ARA′ = 90°, ∠BRB′ = 90°, and so on.
Each angle is the angle of rotation.
We can use a protractor to check.

Check

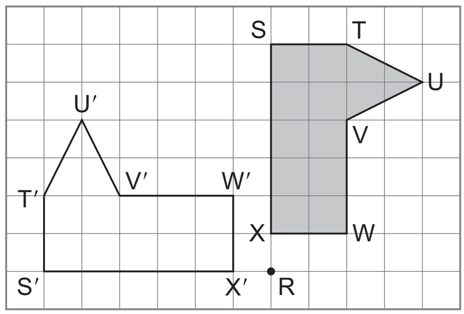
1. For each picture, write the angle of rotation.

a)



Angle of rotation: __________

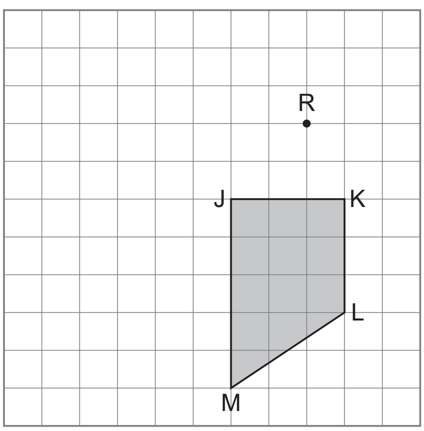
b)



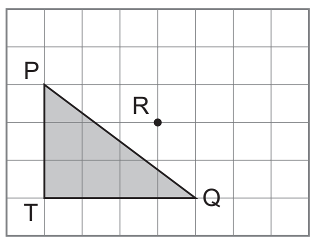
Angle of rotation: ______________________

2. Draw the image after each rotation about point R.

a) 90° clockwise



b) 180°



We can use tracing paper to help us rotate a shape.

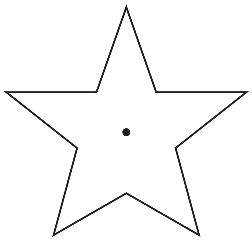
7.6 Rotations and Rotational Symmetry

FOCUS **Draw and classify shapes with rotational symmetry.**

A shape has **rotational symmetry** when it can be turned less than 360° about its centre to match itself exactly.
The number of matches in a complete turn is the **order of rotation.**

Example 1 Determining the Order of Rotational Symmetry

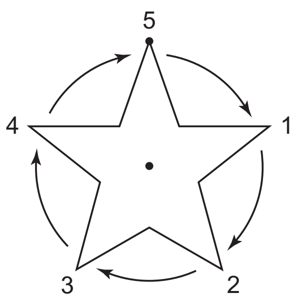
Find the order of rotational symmetry for this star.



Solution

Trace the star.
Draw a dot on the top vertex of each star.
Place the tracing on top of the star so they match exactly.
Rotate the tracing about its centre to see how many times the stars match in one complete turn.
The stars match 5 times.
So, the star has rotational symmetry of order 5.

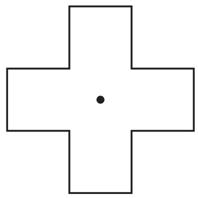
You have made a complete turn when the two dots match again.



Check

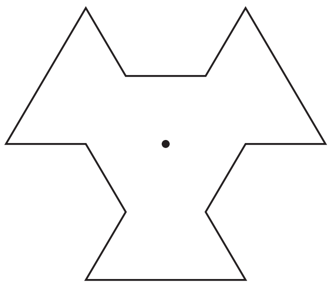
1. Find the order of rotational symmetry for each shape.
Use tracing paper to help.

a)



The shape and its tracing match _____ times.
So, the shape has rotational symmetry of order _____.

b)



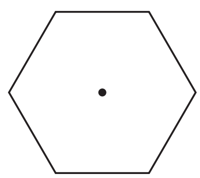
The shape and its tracing match _____ times.
So, the shape has rotational symmetry of order _____.

The smallest angle you need to turn for two shapes to match is the **angle of rotation.**

$$\text{The angle of rotation symmetry} = \frac{360°}{\text{the order of rotation}}$$

Example 2 Determining the Angle of Rotation Symmetry

Find the angle of rotation symmetry for this shape.



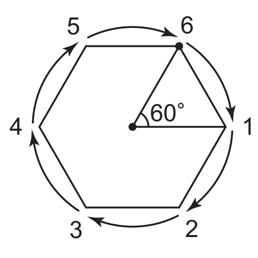
Solution

In one complete turn, the shape and its tracing match 6 times.
So, the order of rotation is 6.

The angle of rotation symmetry is:

$$\frac{360°}{\text{the order of rotation}} = \frac{360°}{6}$$
$$= 60°$$

The angle of rotation symmetry is 60°.

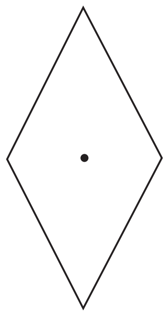


The shapes will match when the tracing is rotated by a multiple of 60°.

Check

1. Find the angle of rotation symmetry for each shape.

a)



The shape and its tracing match ______ times.

So, the order of rotation is ______.

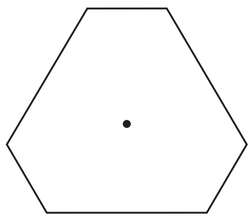
Angle of rotation symmetry is:

$$\frac{360^\circ}{\text{the order of rotation}} = \frac{360^\circ}{\_\_\_\_\_\_}$$

$$= \_\_\_\_\_\_$$

The angle of rotation symmetry is ________.

b)



The shape and its tracing match ______ times.

So, the order of rotation is ______.

Angle of rotation symmetry is:

$$\frac{360^\circ}{\text{the order of rotation}} = \frac{360^\circ}{\_\_\_\_\_\_}$$

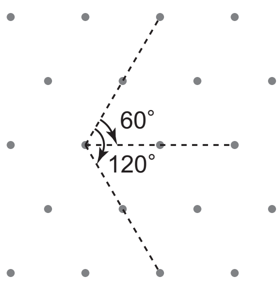
$$= \_\_\_\_\_\_$$

The angle of rotation symmetry is ________.

Shapes that need a complete turn to match again do not have rotational symmetry.

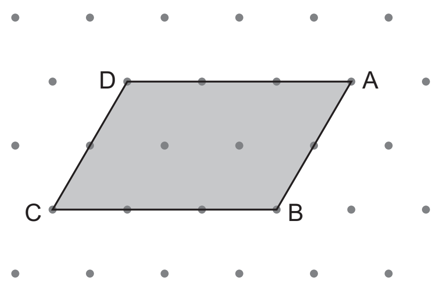
We use isometric dot paper to draw images after rotations that are multiples of 60°.

We can use what we know about isometric dot paper to help us rotate a shape.



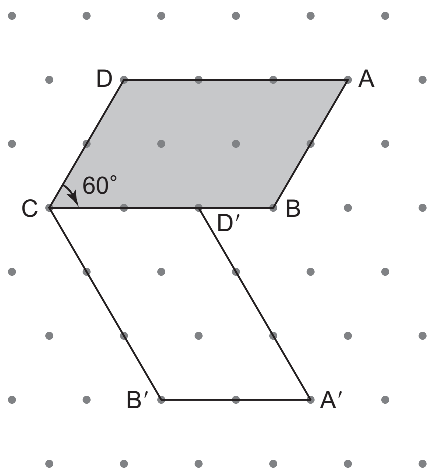
Example 3 Drawing Rotation Images

Rotate parallelogram ABCD 60° clockwise about vertex C.
Draw and label the rotation image.



Solution

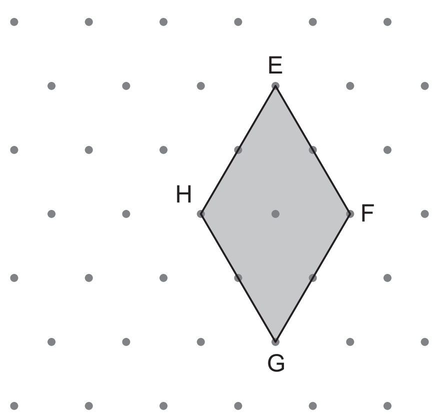
Trace the shape.
Label the vertices on the tracing.
Rotate the tracing 60° clockwise about vertex C.
Draw and label the rotation image.
The centre of rotation, C, does not move.
So, it is not labelled C′.



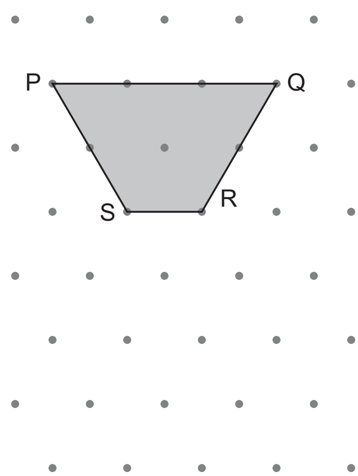
Check

1. Draw and label the image after each rotation.

a) 60° counterclockwise about vertex G



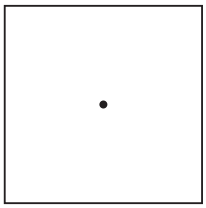
b) 120° clockwise about vertex S



Practice

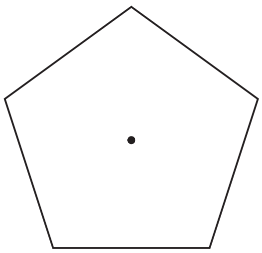
1. Find the order of rotational symmetry for each shape.

a)



The shape and its image match _____ times.
So, the shape has rotational symmetry of order _____.

b)



The shape and its image match _____ times.
So, the shape has rotational symmetry of order _____.

2. Find the angle of rotation symmetry for each shape in question 1.

a) The order of rotation is _____.
Angle of rotation symmetry is:

$$\frac{360°}{\text{the order of rotation}} = \frac{360°}{\underline{\qquad}}$$
$$= \underline{\qquad\quad}$$

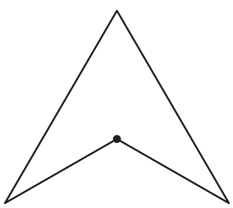
The angle of rotation symmetry is ________.

b) The order of rotation is _____.
Angle of rotation symmetry is:

$$\frac{360°}{\text{the order of rotation}} = \frac{360°}{\underline{\qquad}}$$
$$= \underline{\qquad\quad}$$

The angle of rotation symmetry is ________.

3. Does this shape have rotational symmetry?



____

____

____

4. The angle of rotation symmetry for a shape is 36°.
What is the shape's order of rotation?

The angle of rotation symmetry is: $\dfrac{360°}{\text{the order of rotation}}$

So, $36° = \dfrac{360°}{\text{order of rotation}}$

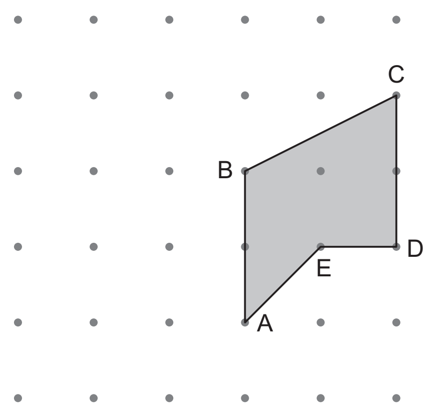
Think: Which number divides into 360 exactly 36 times?

I know 360 ÷ ____ = 36

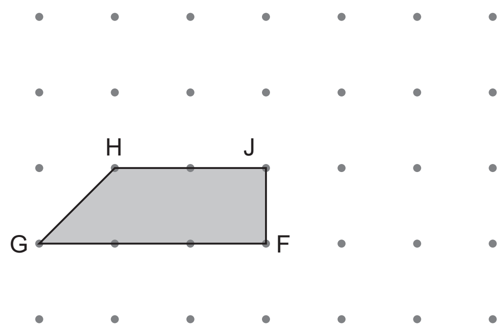
So, the order of rotation is ____.

5. Draw the image after each rotation.

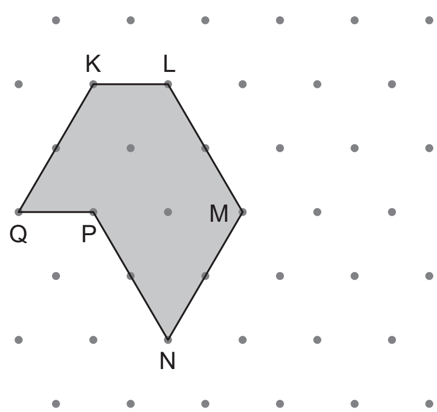
a) 90° counterclockwise about vertex A



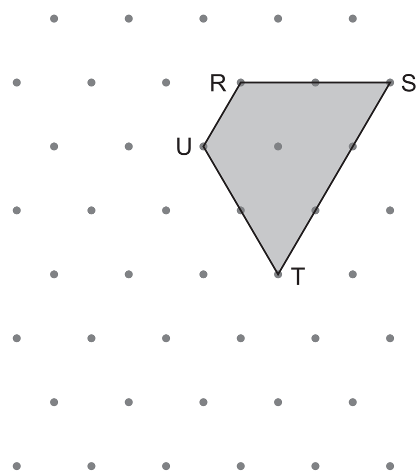
b) 180° about vertex J



c) 60° clockwise about vertex N



d) 120° counterclockwise about vertex T

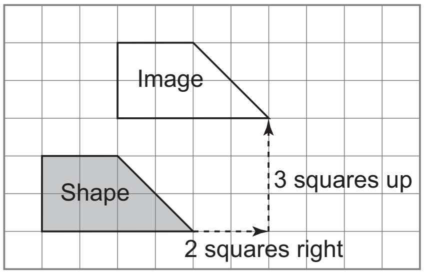


7.7 Skill Builder

Translations

A **translation** moves a shape along a straight line.
A shape and its translation image face the same way.

This shape was translated 2 squares right and 3 squares up.

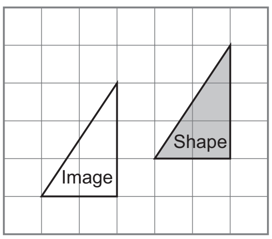


We say how many squares left or right before we say how many up or down.

Check

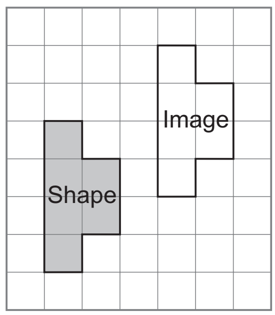
1. Write the translation that moves each shape to its image.

a)



____

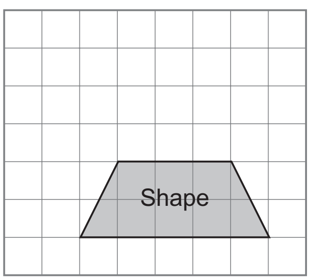
b)



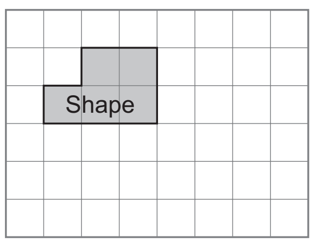
____

2. Draw each translation image.

a) 1 square left and 3 squares up



b) 3 squares right and 2 squares down



7.7 Identifying Types of Symmetry on the Cartesian Plane

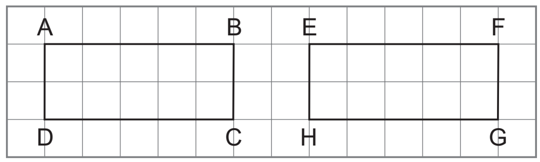
FOCUS **Identify and classify line and rotational symmetry.**

A diagram of a shape and its transformation image may have:

- line symmetry
- rotational symmetry
- both line symmetry and rotational symmetry
- no symmetry

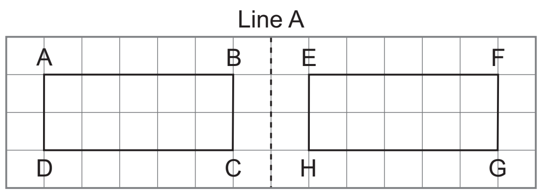
Example 1 Determining whether Shapes Are Related by Symmetry

Are rectangles ABCD and EFGH related by symmetry?



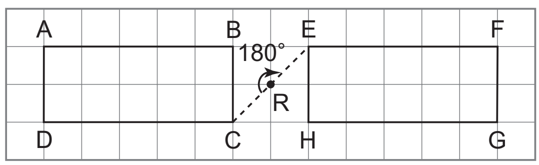
Solution

Check for line symmetry:
Rectangle ABCD is to the left of rectangle EFGH.
So, try a vertical line of reflection.
When I place a Mira on Line A,
the rectangle and its image match.
So, the rectangles are related by line symmetry.



Matching points are the same distance from the line of reflection.

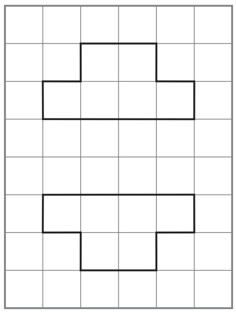
Check for rotational symmetry:
The rectangles do not touch.
So, try a point of rotation off the rectangles.
Try different points to see if the rectangles ever match. When I rotate rectangle ABCD 180° about point R, the rectangles match.
So, the rectangles are related by rotational symmetry.



Check

1. For each diagram, find out if the polygons are related by symmetry.

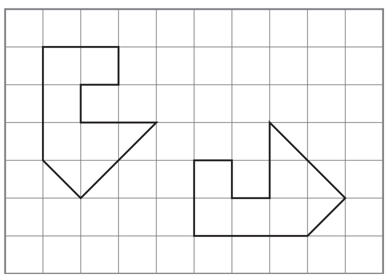
a)



Do the polygons face opposite ways? ________
One polygon is above the other,
so try a ____________ line of reflection.
Use a Mira to find the line of reflection.
Are the polygons related by a reflection? ________
If they are, draw the line of reflection.

Do the polygons touch? ________
So, try a point of rotation ________ the polygons.
Try different points of rotation.
Do the polygons ever match? ________
Are the polygons related by a rotation? ________
If they are, label the point of rotation.

b)

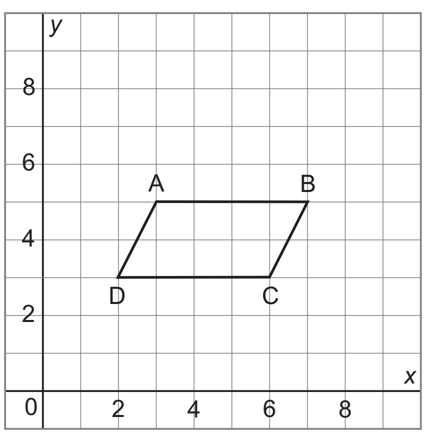


Do the polygons face different ways? ________
Do the polygons face opposite ways? ________
So, are the polygons related by a reflection? ________

Do the polygons touch? ________
So, try a point of rotation ________ the polygons.
Try different points of rotation.
Do the polygons ever match? ________
Are the polygons related by a rotation? ________

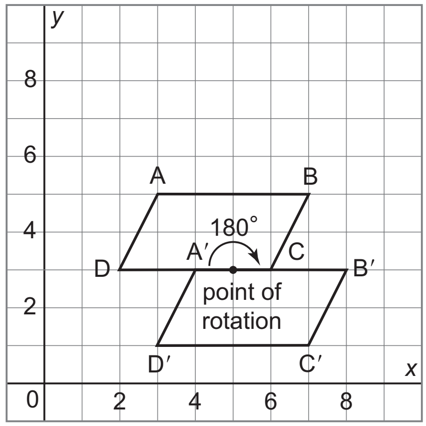
Example 2 Identifying Symmetry in a Shape and Its Transformation Image

Draw the image of this parallelogram after a translation of 2 squares down and 1 square right.
Write the coordinates of each vertex and its image. Describe any symmetry that results.



Solution

Translate parallelogram ABCD 2 squares down
and 1 square right.
Draw and label the translation image.
Write the coordinates of each vertex and its image.



Point	Image
A(3, 5)	A′(4, 3)
B(7, 5)	B′(8, 3)
C(6, 3)	C′(7, 1)
D(2, 3)	D′(3, 1)

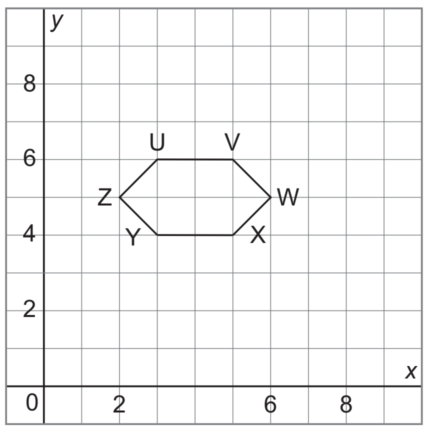
Use a Mira to check for line symmetry.
There is no line on which I can place a Mira
so one parallelogram matches the other.
So, the shape does not have line symmetry.

Use tracing paper to check for rotational symmetry.
The shape and its tracing match after a rotation
of 180° about (5, 3).
So, the shape has rotational symmetry.

Check

1. Draw the image of this polygon after
a translation of 2 squares down.
Write the coordinates of each vertex and its image.
Describe any symmetry that results.

Translate the polygon 2 squares down.
Draw and label the translation image.

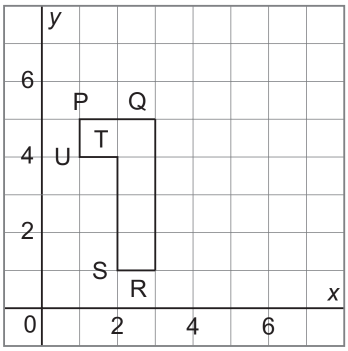


Point	Image
U(3, 6)	Y(3, 4)
V(5, 6)	X(5, 4)
W(___, ___)	W′(___, ___)
X(___, ___)	X′(___, ___)
Y(___, ___)	Y′(___, ___)
Z(___, ___)	Z′(___, ___)

Use a Mira to check for line symmetry.
The shape has _____ lines of symmetry:
Draw and label any lines of symmetry you found.

Use tracing paper to check for rotational symmetry.
Does the shape have rotational symmetry? ________
Draw and label the point of rotation.

2. Draw the image of this polygon after a reflection in the line along side QR.
Write the coordinates of each vertex and its image.
Describe any symmetry that results.



Reflect the polygon.
Draw and label the reflection image.

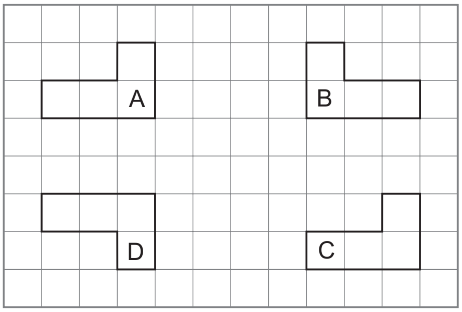
Point	Image
P(___, ___)	P′(___, ___)
Q(___, ___)	Q(___, ___)
R(___, ___)	R(___, ___)
S(___, ___)	S′(___, ___)
T(___, ___)	T′(___, ___)
U(___, ___)	U′(___, ___)

Use a Mira to check for line symmetry.
The shape has _____ line of symmetry:
Draw and label any lines of symmetry you found.

Use tracing paper to check for rotational symmetry.
Is there a point about which you can turn the tracing so it matches the shape? ________
Does the shape have rotational symmetry? ________

Practice

1. Which of these polygons are related by line symmetry?



Which pairs of polygons face opposite ways?

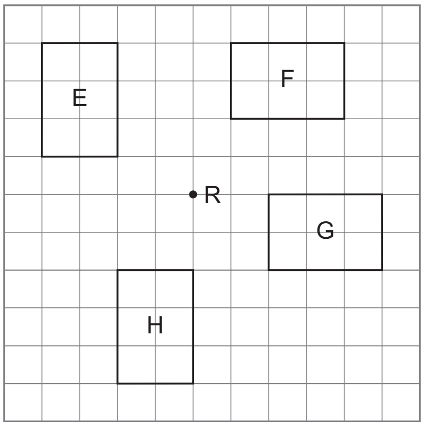
____

Draw in the line of reflection for each pair of polygons.

Which polygons are related by line symmetry?

____

2. Which of these polygons are related by rotational symmetry about point R?



Trace rectangle E.

Rotate the tracing about point R.

Which rectangle does it match? ______________

Trace rectangle G.

Rotate the tracing about point R.

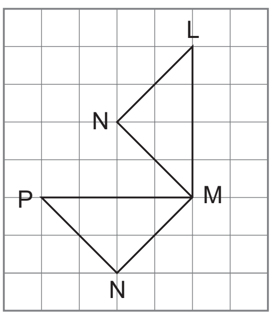
Which rectangle does it match? ______________

Which rectangles are related by rotational symmetry?

____

3. For each diagram, find out if the triangles are related by symmetry.
Use tracing paper and a Mira to help.

a)



Do the triangles face opposite ways? ________

So, are the triangles related by a reflection? ________

Do the triangles touch? ________

So, try a point of rotation ________ the triangles.

Which vertex is common to both triangles?

____

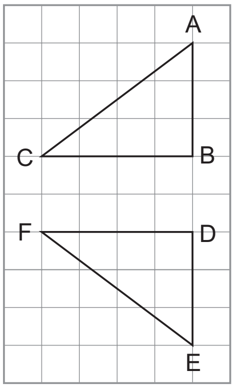
Try different rotations about this vertex.

When do the triangles match? ____________________

____

Are the triangles related by a rotation? _________

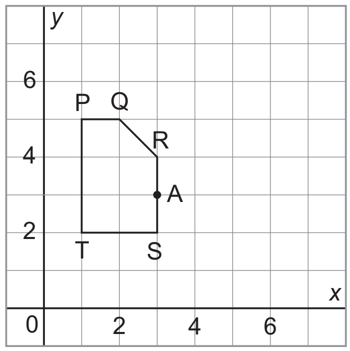
If they are, label the point of rotation.

b) 

Do the triangles face opposite ways? ________
One triangle is above the other,
so try a _____________ line of reflection.
Use a Mira to find the line of reflection.
Are the triangles related by a reflection? ________
If they are, draw the line of reflection.

Do the triangles touch? ________
So, try a point of rotation ________ the triangles.
Try different points of rotation.
Do the triangles ever match? ________
Are the triangles related by a rotation? ________
If they are, label the point of rotation.

4. Draw the image of this polygon after a rotation of 180° about point A.
Write the coordinates of each vertex and its image.
Describe any symmetry that results.



Rotate the polygon.
Draw and label the rotation image.

Point	**Image**
P(___, ___)	P′(___, ___)
Q(___, ___)	Q′(___, ___)
R(___, ___)	S(___, ___)
S(___, ___)	R(___, ___)
T(___, ___)	T′(___, ___)

Use a Mira to check for line symmetry.

____

____

____

Use tracing paper to check for rotational symmetry.
Does the shape have rotational symmetry? ________
If it does, label the point of rotation.

Unit 7 Puzzle

Mystery Logo!

A friend designed a logo for Hal's new gift-wrapping business.
Follow these instructions to create the logo on the coordinate grid below.

Instructions:

1. a) Plot and label the points H(1, 7), A(3, 5), L(1, 3).
Join the points in order to form a triangle. Shade the triangle.

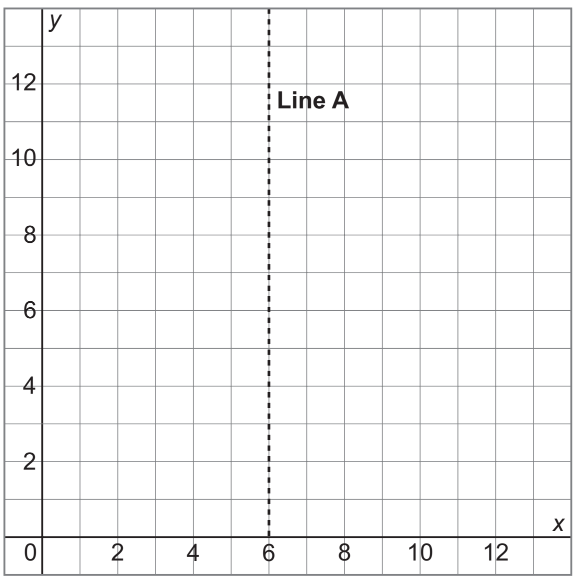
b) Rotate △HAL 90° counterclockwise about H. Shade the triangle.

c) Rotate △HAL 90° clockwise about L. Shade the triangle.

d) Reflect △HAL in the vertical line through A. Shade the triangle.

2. Reflect the shape from part 1 in Line A.
Shade to match the shape in part 1.

3. Plot the points (5, 6), (7, 6), (7, 4), (5, 4).
Join the points in order to form a square. Shade the square a different colour.



Does the logo have any symmetry?

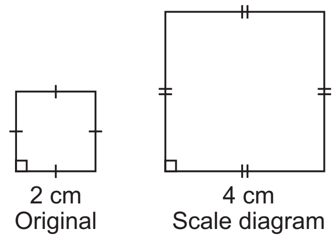
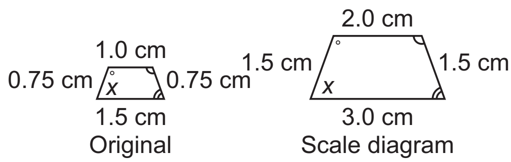
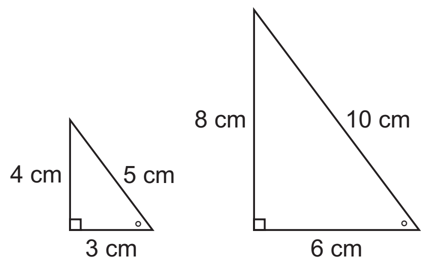
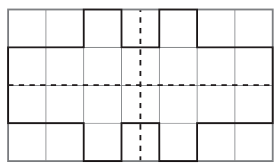
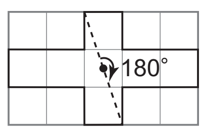
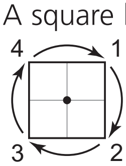
____

____

____

____

Unit 7 Study Guide

Skill	Description	Example
Find the scale factor of a scale diagram.	Scale factor $= \frac{\text{length on scale diagram}}{\text{length on original diagram}}$ An enlargement has a scale factor > 1. A reduction has scale a factor < 1.	 Scale factor: $\frac{\text{length on scale diagram}}{\text{length on original diagram}} = \frac{4}{2} = 2$
Find out if two polygons are similar.	In two similar polygons: – matching angles are equal *and* – all pairs of matching sides have the same scale factor.	
Find out if two triangles are similar.	In two similar triangles: – matching angles are equal *or* – all pairs of matching sides have the same scale factor.	
Identify lines of symmetry.	A line of symmetry divides a shape into 2 congruent parts. When one part is reflected in the line of symmetry, it matches the other part exactly.	
Find out if a shape has rotational symmetry.	A shape has rotational symmetry when it can be turned less than 360° about its centre to match itself exactly.	
Find the order of rotation and the angle of rotation symmetry for a polygon.	The number of times a shape matches itself in one complete turn is the order of rotation. The angle of rotation symmetry is: $\frac{360°}{\text{the order of rotation}}$	A square has order of rotation 4.  So, its angle of rotation symmetry is: $\frac{360°}{4} = 90°$

Unit 7 Review

7.1 **1.** A photo of a baby giraffe is to be enlarged for a newspaper.
The actual photo measures 4 cm by 6 cm.

Find the dimensions of the enlargement with a scale factor of $\frac{7}{2}$.

Write the scale factor as a decimal: $\frac{7}{2}$ = ________

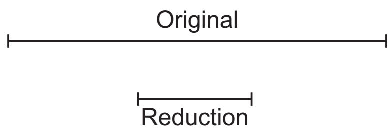
Length of original photo: ________
Length of enlargement: ________ × ________ = ________

Width of original photo: ________
Width of enlargement: ________ × ________ = ________

The enlargement has dimensions ____________________.

7.2 **2.** Find the scale factor for this reduction.

Length of original line segment: ________ cm
Length of reduction: ________ cm



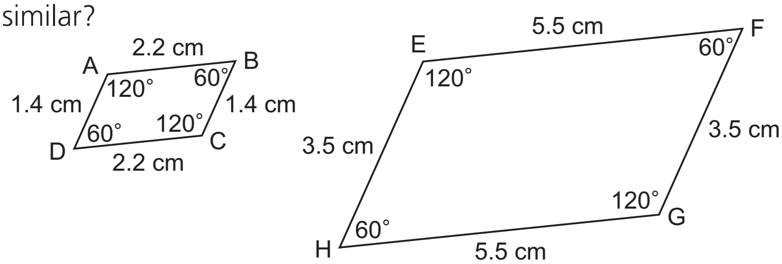
$$\text{Scale factor} = \frac{\text{length on reduction}}{\text{length on original}}$$

$$= \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$$

$$= \_\_\_\_\_\_$$

The scale factor is ______.

7.3 **3.** Are these parallelograms similar?



Check matching angles.

∠A = _____ = ______   ∠B = _____ = _____   ________________   ________________

All matching angles ______ equal.

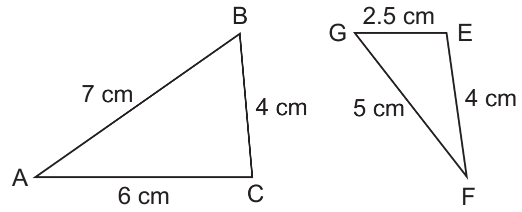
Check matching sides.
The matching sides are: ______ and ______, and ______ and ______. Find the scale factors.

$$\frac{\text{length of } \_\_\_\_\_}{\text{length of } \_\_\_\_\_} = \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_} \qquad \frac{\text{length of } \_\_\_\_\_}{\text{length of } \_\_\_\_\_} = \frac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$$

$$= \_\_\_\_\_\_ \qquad = \_\_\_\_\_\_$$

The scale factors ______ equal. So, the parallelograms ______ similar.

7.4 **4.** Are these two triangles similar?



In △ABC, order the sides from shortest to longest:

____

In △EFG, order the sides from shortest to longest:

____

Find the scale factors of matching sides.

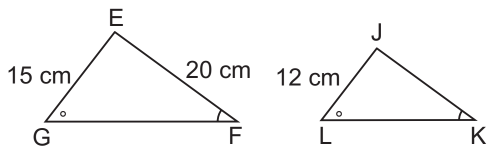
$\frac{\text{length of } \_\_\_\_\_}{\text{length of } \_\_\_\_\_} = \frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_} = \_\_\_\_\_\_\_$

$\frac{\text{length of } \_\_\_\_\_}{\text{length of } \_\_\_\_\_} = \frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_} = \_\_\_\_\_\_\_$

$\frac{\text{length of } \_\_\_\_\_}{\text{length of } \_\_\_\_\_} = \frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_} = \_\_\_\_\_\_\_$

All scale factors are ____________. So, the triangles ________________.

5. Triangle EFG is similar to △JKL.
Find the length of JK.



________ is a reduction of ________.

Choose a pair of matching sides whose lengths are both known:

____

$$\text{Scale factor} = \frac{\text{length on reduction}}{\text{length on original}}$$

$$= \frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_}$$

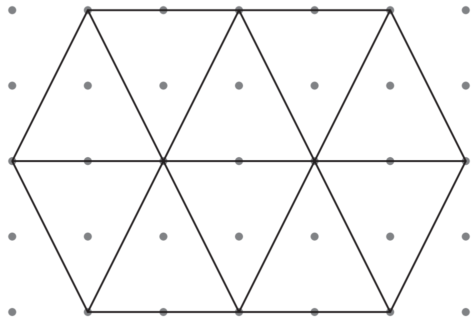
$$= \_\_\_\_\_\_\_\_$$

The scale factor is ________.

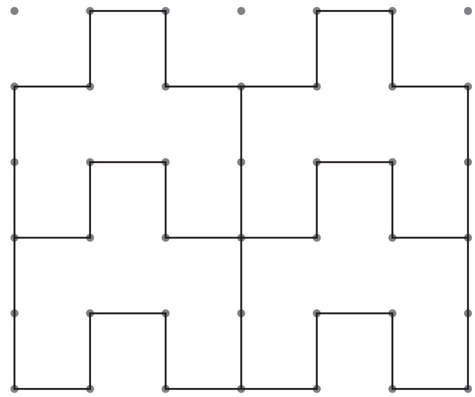
Use the scale factor to find the length of JK.
JK and EF are matching sides.
Length of EF: _________
Scale factor: _________
Length of JK: ________________________
So, JK has length _________.

7.5 **6.** Draw the lines of symmetry in each tessellation.

a)

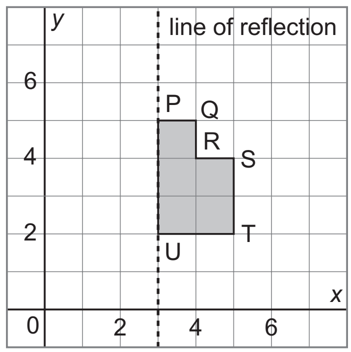


b)

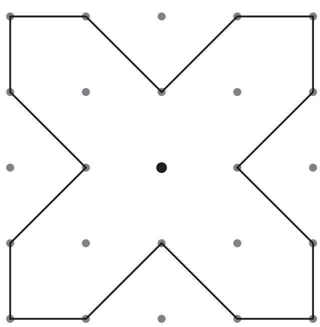


7. Reflect the shape in the line of reflection to make a larger shape.

Point	Image
P(___, ___)	________
Q(___, ___)	________
R(___, ___)	________
S(___, ___)	________
T(___, ___)	________
U(___, ___)	________



7.6 **8.** Find the order of rotational symmetry and the angle of rotation symmetry for this shape.



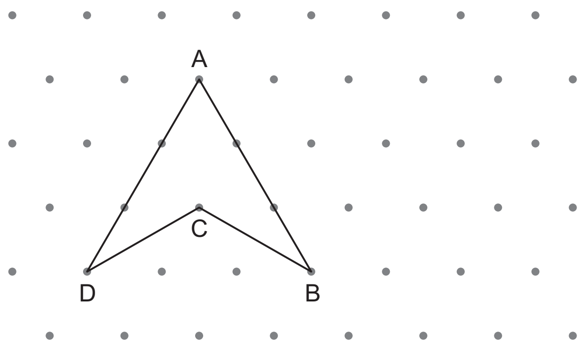
The shape and its image match ____ times.
So, the shape has rotational symmetry of order ____.
Angle of rotation symmetry is:

$$\frac{360^\circ}{\text{the order of rotation}} = \frac{360^\circ}{\_\_\_\_\_\_}$$

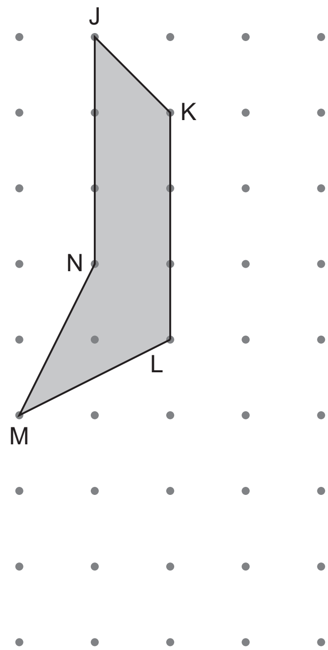
$$= \_\_\_\_\_\_$$

9. Draw the image after each rotation.

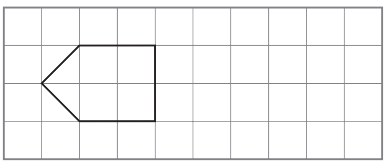
a) 120° clockwise about vertex B



b) 180° about vertex L



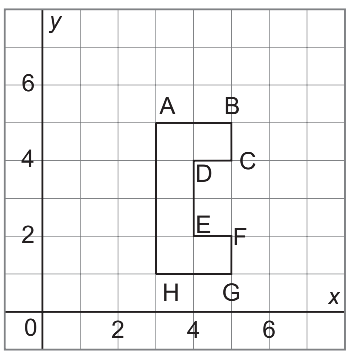
7.7 **10.** Find out if the polygons are related by symmetry.
Use tracing paper and a Mira to help.



Do the polygons face opposite ways? _________
So, are the polygons related by a reflection? _________
Draw and label the line of reflection.

Do the polygons touch? _________
So, try a point of rotation _________ the polygons.
Are the polygons related by a rotation? _________
If they are, label the point of rotation.

11. a) Reflect the polygon in the vertical line through 3 on the *x*-axis.
Draw and label the image.



b) Describe the symmetry in the shape that results.

The shape has _____ lines of symmetry:
Draw and label any lines of symmetry you found.
Does the shape have rotational symmetry? _________
If it does, label the point of rotation.