UNIT 8

# Circle Geometry

## What You'll Learn

How to

- Solve problems involving tangents to a circle
- Solve problems involving chords of a circle
- Solve problems involving the measures of angles in a circle

## Why Is It Important?

Circle properties are used by

- artists, when they create designs and logos

## Key Words

| | |
|---|---|
| radius (radii) | chord |
| right angle | perpendicular bisector |
| tangent | central angle |
| point of tangency | inscribed angle |
| diameter | arc |
| right triangle | subtended |
| isosceles triangle | semicircle |

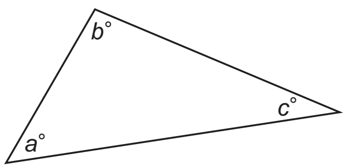
# 8.1 Skill Builder

## Solving for Unknown Measures in Triangles

Here are 2 ways to find unknown measures in triangles.

**Angle Sum Property**

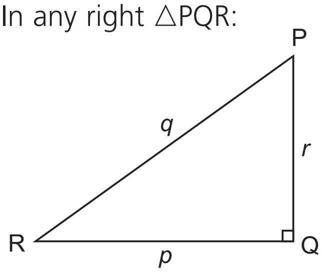
In any triangle:




$a° + b° + c° = 180°$

**Pythagorean Theorem**

In any right △PQR:



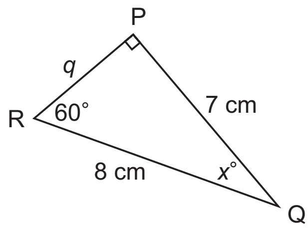

$q^2 = p^2 + r^2$

Here is how to find the unknown measures in right △PQR.

In △PQR, the angles add up to 180°.
To find $x°$, start at 180° and subtract the known measures.

$x° = 180° - 90° - 60°$

$= 30°$




By the Pythagorean Theorem:

$QR^2 = PR^2 + PQ^2$

$8^2 = q^2 + 7^2$

So: $q^2 = 8^2 - 7^2$

$q = \sqrt{8^2 - 7^2}$

$\doteq 3.87$

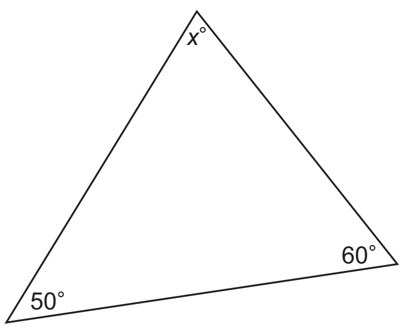
So, $x°$ is 30° and $q$ is about 4 cm.

*Answer to the same degree of accuracy as the question uses.*

## *Check*

**1.** Find each unknown measure.

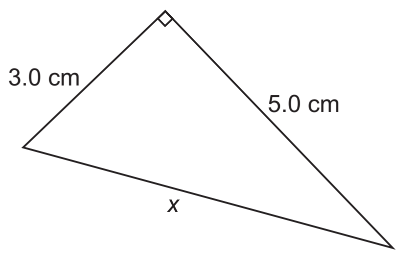
**a)**




$x° = 180° - $ ______ $-$ ______

$=$ ______

**b)**




$x^2 =$ ______ $+$ ______

$x = \sqrt{\_\_\_\_\_\_ + \_\_\_\_\_\_}$

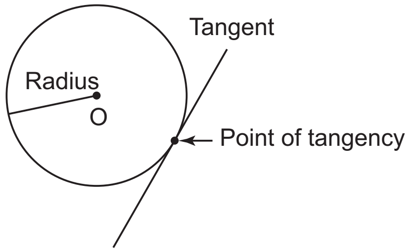
$\doteq$ ______

So, $x$ is ________________.

# 8.1 Properties of Tangents to a Circle

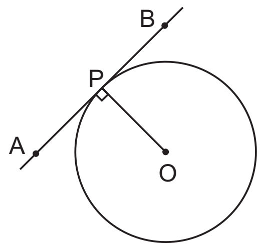
**FOCUS** **Use the relationship between tangents and radii to solve problems.**

A **tangent** touches a circle at exactly one point.




## Tangent-Radius Property

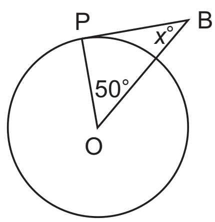
A tangent to a circle is perpendicular to the radius drawn to the point of tangency.
So, OP ⊥ AB, ∠OPA = 90° and ∠OPB = 90°




*⊥ means "perpendicular to".*

## *Example 1* Finding the Measure of an Angle in a Triangle

BP is tangent to the circle at P.
O is the centre of the circle.
Find the measure of $x°$.




### *Solution*

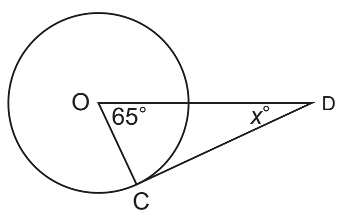
By the tangent-radius property: ∠OPB = 90°
Since the sum of the angles in △OPB is 180°:
$x° = 180° - 90° - 50°$
$= 40°$
So, $x°$ is 40°.

### *Check*

**1.** Find the value of $x°$.

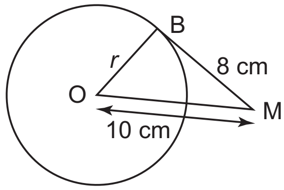
∠_______ = 90°

$x°$ = 180° − _____ − _____
= _____



## *Example 2* Using the Pythagorean Theorem in a Circle

MB is a tangent to the circle at B. O is the centre.
Find the length of radius OB.




### *Solution*

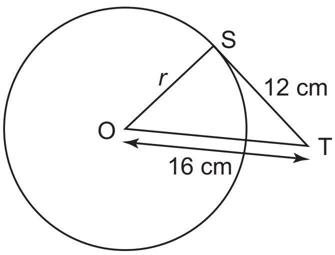
By the tangent-radius property: $\angle OBM = 90°$
By the Pythagorean Theorem in right $\triangle MOB$:

$$OM^2 = OB^2 + BM^2$$
$$10^2 = r^2 + 8^2$$
$$100 = r^2 + 64$$
$$100 - 64 = r^2$$
$$36 = r^2$$
$$\sqrt{36} = r$$
$$r = 6$$

Radius OB has length 6 cm.

### *Check*

**1.** ST is a tangent to the circle at S. O is the centre.
Find the length of radius OS.
Answer to the nearest millimetre.




$\angle OST =$ ______ By the tangent-radius property

$OT^2 =$ ______ + ______ By the Pythagorean Theorem

______ $= r^2 +$ ______

______ $= r^2 +$ ______

______ − ______ $= r^2$

______ $= r^2$

$\sqrt{\text{______}} = r$

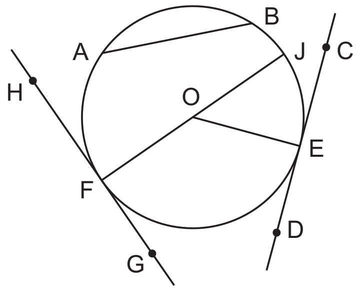
$r \doteq$ ______

OS is about ______ cm long.

## Practice

In each question, O is the centre of the circle.

**1.** From the diagram, identify:




**a)** 3 radii _______, _______, _______

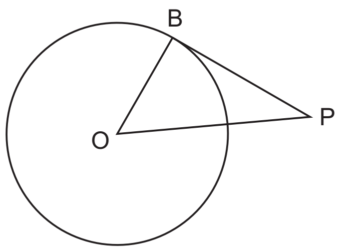
**b)** 2 tangents _______, _______

**c)** 2 points of tangency _______, _______

**d)** 4 right angles ∠_______, ∠_______, ∠_______, ∠_______

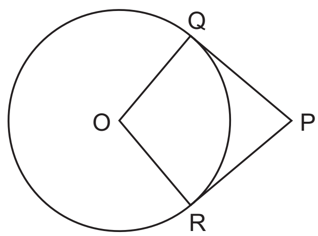
**2.** What is the measure of each angle?

**a)**




∠OBP = _____

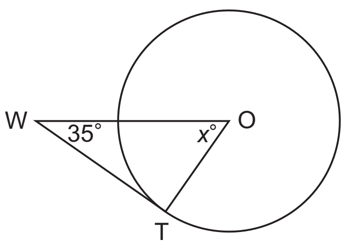
**b)**




∠PQO = _____ ∠PRO = _____

**3.** Find each value of $x°$.

**a)**

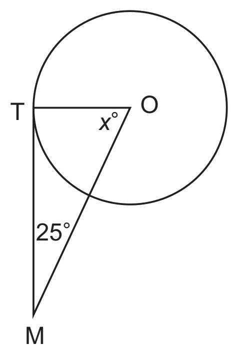



∠OTW = ______

$x° = 180° -$ ______ $-$ ______

= ______

**b)**

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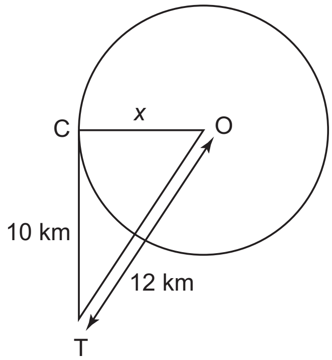
____________________________

= ______

**4.** Find each value of $x$.

Answer to the nearest tenth of a unit.

**a)**




$\angle OCT = 90°$ By the tangent-radius property

______ $= x^2 +$ ______ By the Pythagorean Theorem in $\triangle OCT$

______ $= x^2 +$ ______

______ $-$ ______ $= x^2$

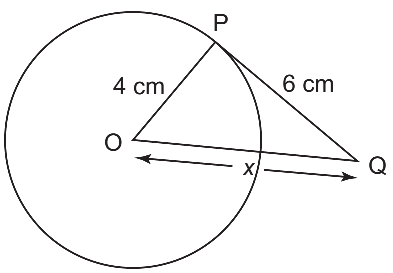
_____ $= x^2$

$\sqrt{\_\_\_\_\_\_} = x$

$x \doteq$ _____

So, OC is about _____ km.

**b)**




$\angle OPQ =$ ______, and:

$x^2 =$ ______ $+$ ______

$x^2 =$ ______ $+$ ______

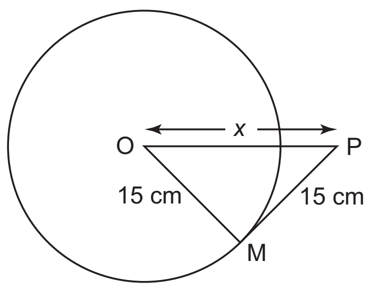
$x^2 =$ ______

$x = \sqrt{\_\_\_\_\_\_}$

$x \doteq$ ______

So, OQ is about ______ cm.

**c)**




$x^2 =$ ______ $+$ ______

$x^2 =$ ______ $+$ ______

$x^2 =$ ______

$x = \sqrt{\_\_\_\_\_\_}$

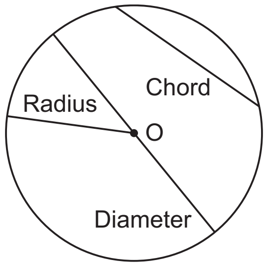
$x \doteq$ ______

So, OP is about ______ cm.

# 8.2 Properties of Chords in a Circle

**FOCUS** **Use chords and related radii to solve problems.**

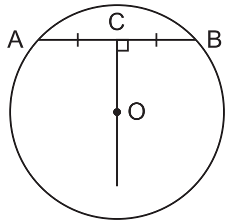
A **chord** of a circle joins 2 points on the circle.




## Chord Properties

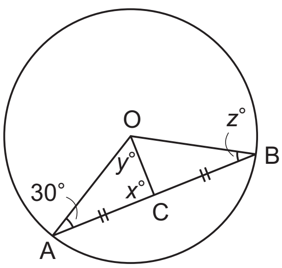
In any circle with centre O and chord AB:

- If OC bisects AB, then OC ⊥ AB.
- If OC ⊥ AB, then AC = CB.
- The perpendicular bisector of AB goes through the centre O.




## *Example 1* Finding the Measure of Angles in a Triangle

Find $x°$, $y°$, and $z°$.

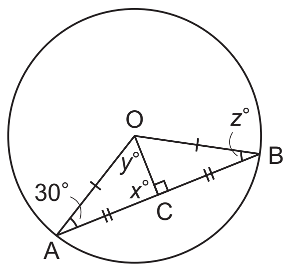



### *Solution*

OC bisects chord AB, so OC ⊥ AB
Therefore, $x° = 90°$

By the angle sum property in △OAC:
$y° = 180° - 90° - 30°$
$= 60°$
Since radii are equal, OA = OB, and △OAB is isosceles.
∠OBA = ∠OAB

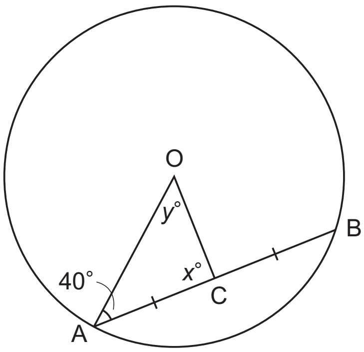
So, $z° = 30°$




*In an isosceles triangle, 2 base angles are equal.*

## *Check*

**1.** Find the values of $x°$ and $y°$.

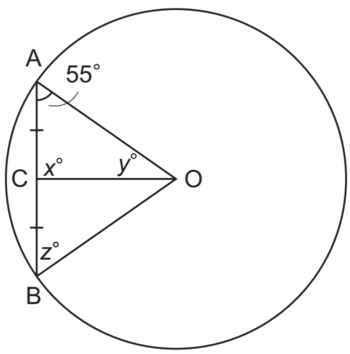



_____ ⊥ _____ So, $x°$ = _____ By the chord properties

$y°$ = _____ − _____ − _____ By the angle sum property

= _____

**2.** Find the values of $x°$, $y°$, and $z°$.




_____ ⊥ _____ So, $x°$ = _____ By the chord properties

$y°$ = _____ − _____ − _____ By the angle sum property

= _____

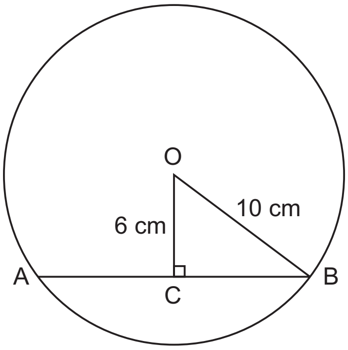
Since OA = _____,

△_____ is isosceles and ∠_____ = ∠_____

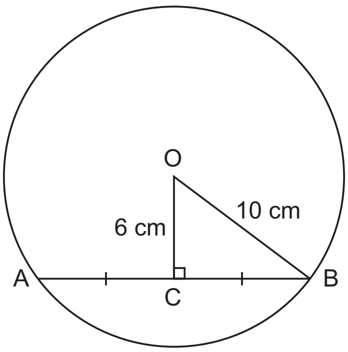
So, $z°$ = _____

## *Example 2* Using the Pythagorean Theorem in a Circle

O is the centre of the circle.
Find the length of chord AB.




### *Solution*




$10^2 = 6^2 + BC^2$ By the Pythagorean Theorem in right △OCB

$100 = 36 + BC^2$

$100 - 36 = BC^2$

$64 = BC^2$

$BC = \sqrt{64}$

$= 8$

So, BC = 8 cm

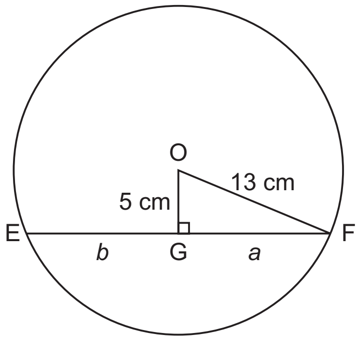
Since OC ⊥ AB, OC bisects AB. By the chord properties

So, AC = BC = 8 cm

The length of chord AB is: 2 × 8 cm = 16 cm

## *Check*

**1.** Find the values of *a* and *b*.




_____ = _____ + $a^2$ By the Pythagorean Theorem in right △OFG

__________________

__________________

__________________

__________________

__________________

So, $a$ = _____ cm

_____ = _____ By the chord properties

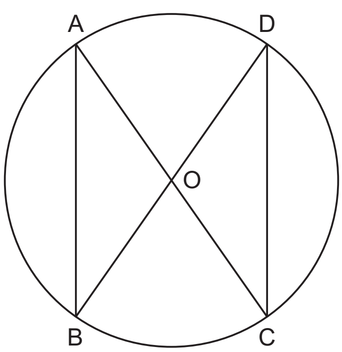
So, $b$ = _____ cm

## Practice

In each diagram, O is the centre of the circle.

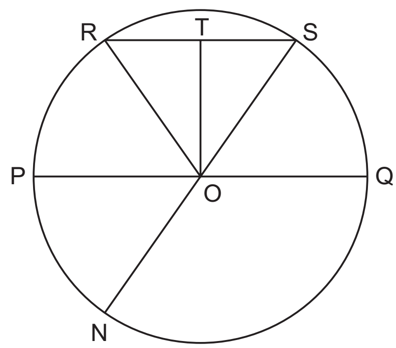
**1.** Name all radii, chords, and diameters.

**a)**




Radii: ______________________
Chords: ______________________
Diameters: ______________________

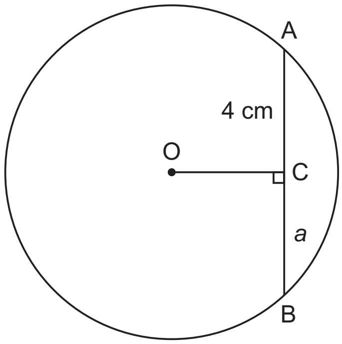
**b)**




Radii: ______________________
Chords: ______________________
Diameters: ______________________

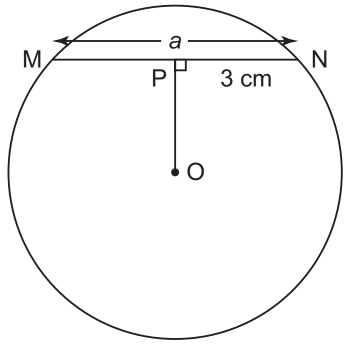
**2.** On each diagram, mark line segments with equal lengths.
Then find each value of *a*.

**a)**



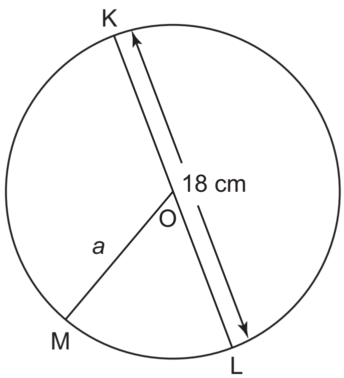

AC = CB = ______ cm
So, $a$ = ______ cm

**b)**



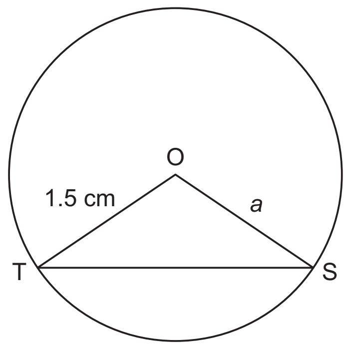

MN = 2 × ______
= 2 × ______ cm
= ______ cm
So, $a$ = ______

**c)**




OL = $\frac{1}{2}$ × ______
= $\frac{1}{2}$ × ______ cm
= ______ cm
So, $a$ = ______

**d)**

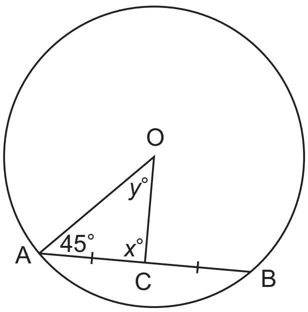



OS = ______ = ______ cm

So, $a$ = ______

**3.** Find each value of $x°$ and $y°$.

**a)**

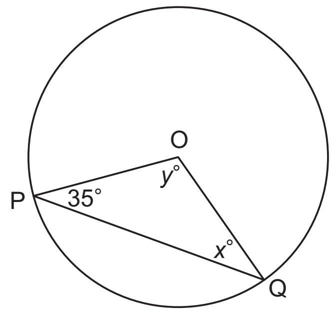



$x° =$ ______

$y° = 180° -$ ______ $-$ ______

$=$ ______

**b)**




______ = ______ △OPQ is ________________

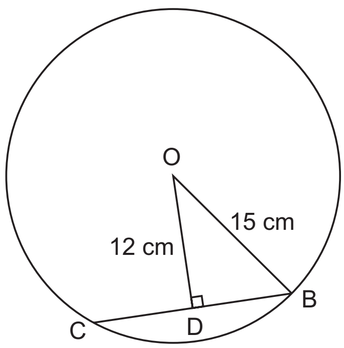
∠______ = ∠______

So, $x° =$ ______

$y° = 180° -$ ______ $-$ ______

$=$ ______

**4.** Find the length of chord BC.




______ = ______ + $DB^2$ By the Pythagorean Theorem

______ = ______ + $DB^2$

____________________

____________________

____________________

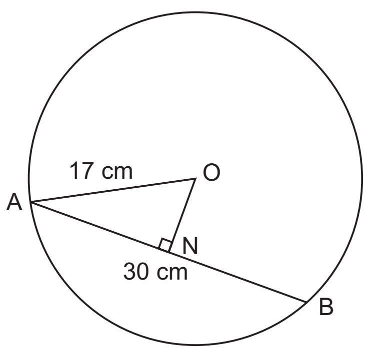
____________________

So, DB = ______ cm

______ = ______ = ______ cm By the chord properties

So, chord BC has length: $2 \times$ ______ cm = ______ cm

**5.** Find ON.




$AN = \frac{1}{2} \times$ ______

$= \frac{1}{2} \times$ ______ cm By the chord properties

$=$ ______ cm

______ = ______ + $ON^2$ By the Pythagorean Theorem

______ = ______ + $ON^2$

____________________

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So, ON is ______ cm.

CHECKPOINT

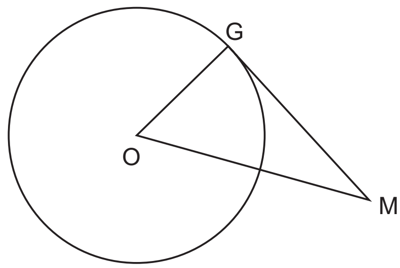
## *Can you ...*

- Solve problems using tangent properties?
- Solve problems using chord properties?

**8.1** In each diagram, O is the centre of the circle.
Assume that lines that appear to be tangent are tangent.

**1.** Name the angles that measure 90°.

**a)**

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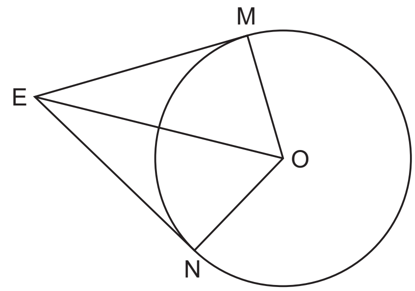
**b)**

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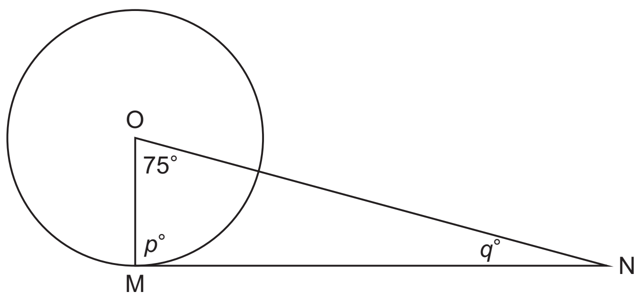
**2.** Find the unknown angle measures.

**a)**

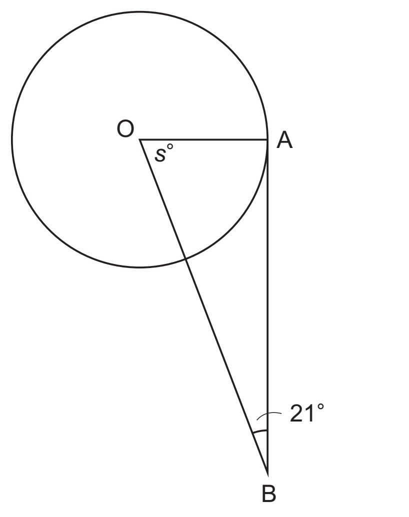



$p° =$ ______ Tangent-radius property

$q° = 180° -$ ______ $-$ ______ Angle sum property

$q° =$ ______

**b)**



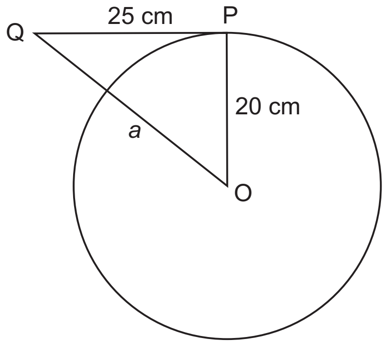

_______ = 90°

$s° =$ ______ $-$ ______ $-$ ______

$s° =$ ______

**3.** Find the values of *a* and *b* to the nearest tenth.

**a)**




∠OPQ = ______ By the tangent-radius property

OQ is ________________ of △OPQ.

$a^2$ = ______ + ______ By the Pythagorean Theorem

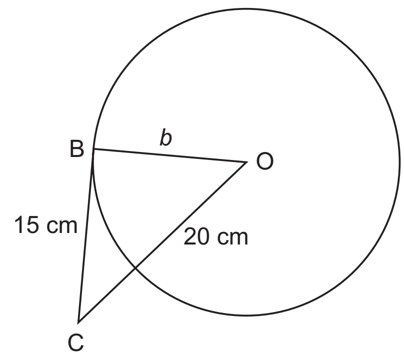
______________________

______________________

______________________

So, $a \doteq$ ______ cm

**b)**




∠OBC = ______

OB is _______ of △OBC.

______ = $b^2$ + ______

______________________

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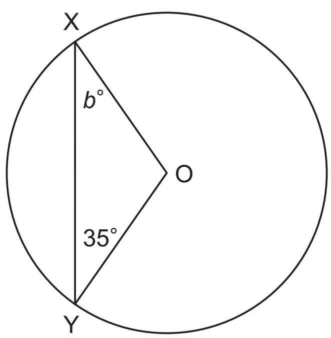
______________________

So, $b \doteq$ ______ cm

**8.2** **4.** Find the unknown measures.

**a)**

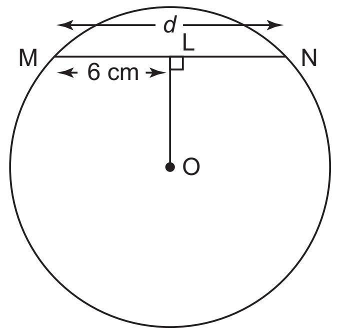



OX = ______

△OXY is ___________.

So, b° = ______

**b)**



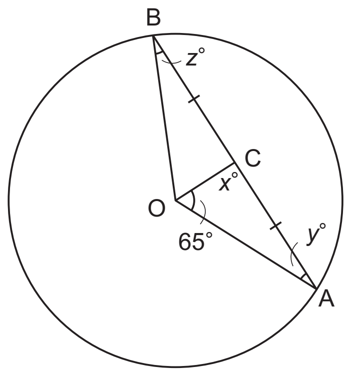

MN = 2 × ______

MN = ______ × ______ cm

= ______ cm

So, *d* = ______ cm

**5.** Find each value of $x°$, $y°$, and $z°$.




$x° =$ ______ By the chord properties

$y° =$ ______ − ______ − ______ By the angle sum property

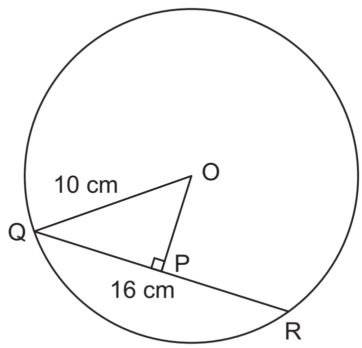
$=$ ______

______ = ______, so ______ is isosceles.

∠______ = ∠______

So, $z° =$ ______

**6.** Find the length of OP.




$QP = \frac{1}{2} \times QR$ By the chord properties

$= \frac{1}{2} \times$ ______ cm

$=$ ______ cm

$OQ^2 =$ ______ $+ OP^2$ By the Pythagorean Theorem

______ = ______ $+ OP^2$

____________________

____________________

____________________

____________________

$=$ ______

So, the length of OP is ______ cm.

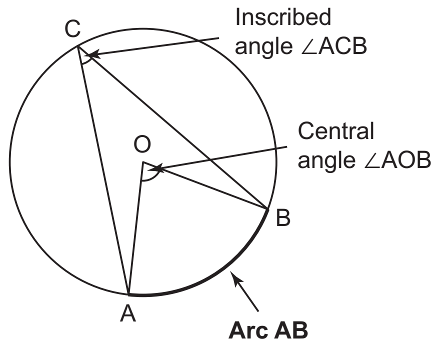
# 8.3 Properties of Angles in a Circle

**FOCUS** **Use inscribed angles and central angles to solve problems.**

In a circle:

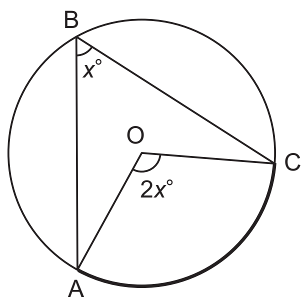
- A **central angle** has its vertex at the centre.
- An **inscribed angle** has its vertex on the circle.

Both angles in the diagram are **subtended** by **arc** AB.




## Central Angle and Inscribed Angle Property

The measure of a central angle is twice the measure of an inscribed angle subtended by the same arc.

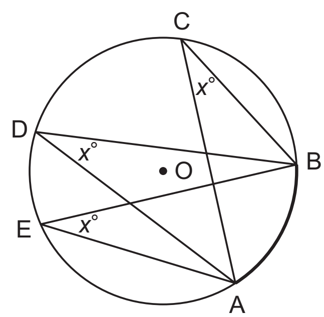



So, $\angle AOC = 2\angle ABC$, or

$\angle ABC = \frac{1}{2}\angle AOC$

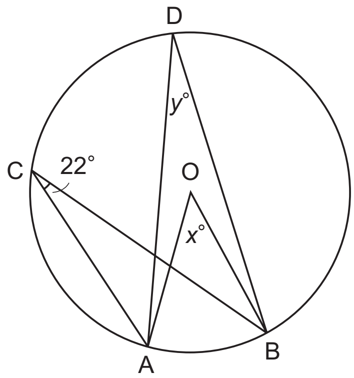
## Inscribed Angles Property

Inscribed angles subtended by the same arc are equal.
So, $\angle ACB = \angle ADB = \angle AEB$

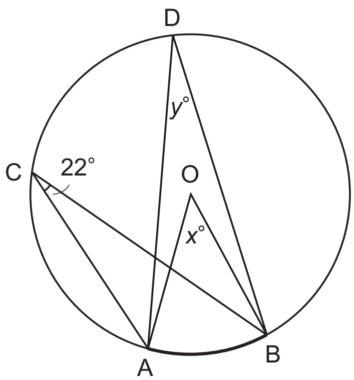


## *Example 1* Using Inscribed and Central Angles

Find the values of $x°$ and $y°$.




### *Solution*




Central ∠AOB and inscribed ∠ACB are both subtended by arc AB.

So, ∠AOB = 2∠ACB

$x° = 2 \times 22°$

$= 44°$

∠ACB and ∠ADB are inscribed angles subtended by the same arc AB.

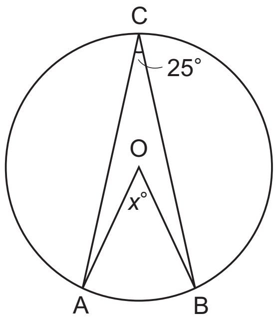
So, ∠ADB = ∠ACB

$y° = 22°$

## *Check*

**1.** Find each value of $x°$.

**a)**

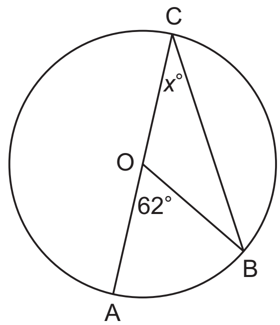



$\angle AOB = 2 \times \angle ACB$

$x° = 2 \times$ ______

$=$ ______

**b)**

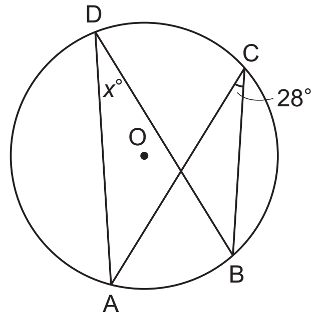



$\angle ACB = \frac{1}{2} \times$ ________

$x° = \frac{1}{2} \times$ ______

$=$ ______

**c)**

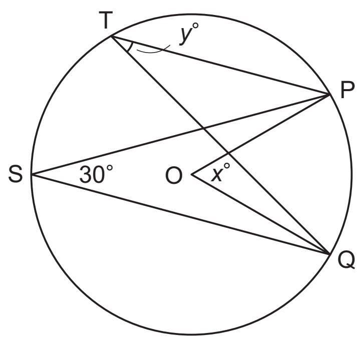



$\angle ADB =$ ________

$x° =$ ______

**2.** Find the values of $x°$ and $y°$.

**a)**




$\angle QOP = 2 \times \angle QSP$

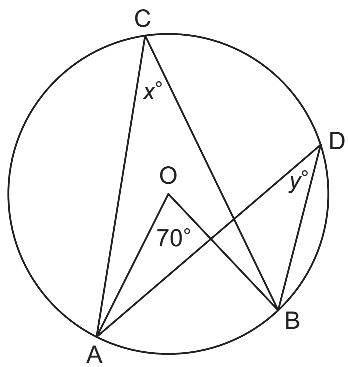
$x° = 2 \times$ ______

$x° =$ ______

$\angle QTP =$ ________

$y° =$ ______

**b)**




$\angle ACB = \frac{1}{2} \times$ ________

$x° = \frac{1}{2} \times$ ______

$x° =$ ______

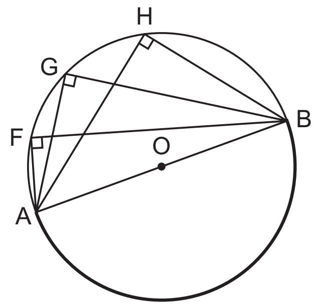
$\angle ADB =$ ________

$y° =$ ______

## Angles in a Semicircle Property

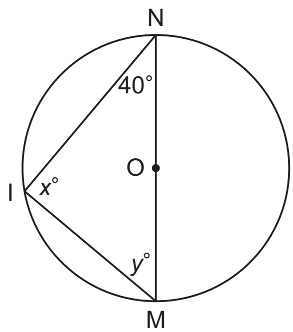
Inscribed angles subtended by a semicircle are right angles.

$\angle AFB = \angle AGB = \angle AHB = 90°$




## *Example 2* Finding Angles in an Inscribed Triangle

Find $x°$ and $y°$.




### *Solution*

$\angle MIN$ is an inscribed angle subtended by a semicircle.

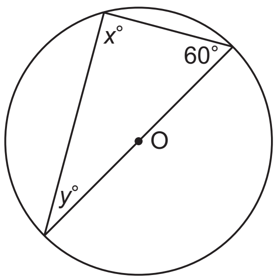
So, $x° = 90°$

$y° = 180° - 90° - 40°$ By the angle sum property in $\triangle MIN$

$= 50°$

### *Check*

**1.** Find the values of $x°$ and $y°$.



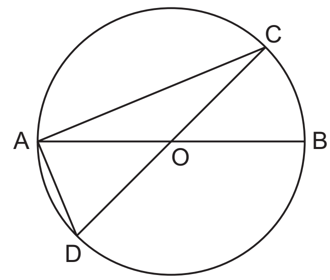

$x°$ = ______

$y° = 180° -$ ______ $-$ ______

$y°$ = ______

## Practice

**1.** Name the following from the diagram.




**a)** the central angle subtended by arc CB: ∠_______

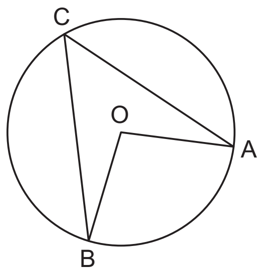
**b)** the central angle and inscribed angle subtended by arc AD: ∠_______ and ∠_______

**c)** the inscribed angle subtended by a semicircle: ∠_______

**d)** the right angle: ∠_______

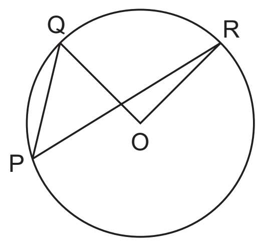
**2.** In each circle, name a central angle and an inscribed angle subtended by the same arc. Shade the arc.

**a)**




Central angle: ∠_______
Inscribed angle: ∠_______

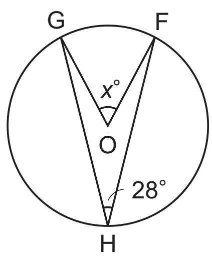
**b)**




Central angle: ∠_______
Inscribed angle: ∠_______

**3.** Determine each indicated measure.

**a)**

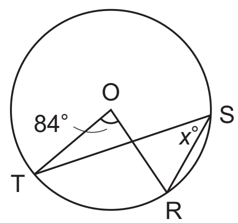



∠GOF = 2 × ∠GHF

$x°$ = 2 × _______

= _______

**b)**

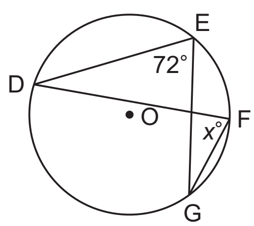



∠TSR = $\frac{1}{2}$ × ∠_______

$x°$ = $\frac{1}{2}$ × _______

$x°$ = _______

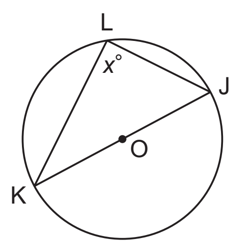
**c)**




∠DEG = _______

$x°$ = _______

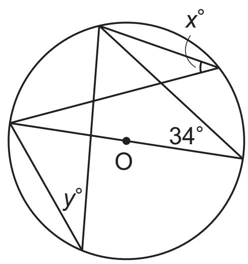
**d)**




$x°$ = _______

**4.** Determine each value of $x°$ and $y°$.

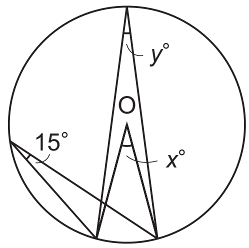
**a)**




$x° =$ _______

$y° =$ _______

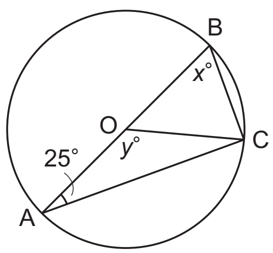
**b)**




$x° =$ _______ $\times$ _______ $=$ _______

$y° =$ _______

**5.** Find the value of $x°$ and $y°$.




$\angle ACB =$ _______

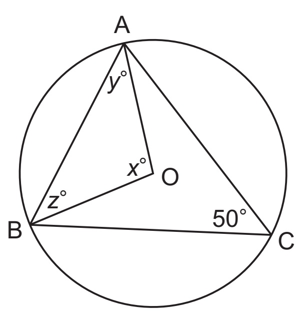
$x° = 180° -$ _______ $-$ _______ By the angle sum property

$=$ _______

$y° =$ _______ $\times$ _______

$=$ _______

**6.** Find the value of $x°$, $y°$, and $z°$.




$\angle AOB = 2 \times$ _______

$x° = 2 \times$ _______ $=$ _______

In $\triangle OAB$, _______ $=$ _______

$\triangle OAB$ is _______________.

In $\triangle OAB$:

$y° = z°$

$y° + y° =$ _______ $-$ _______ By the angle sum property

$2y° =$ _______

$y° = \frac{\_\_\_}{2}$

So, $y° =$ _______ and $z° =$ _______

# Unit 8 Puzzle

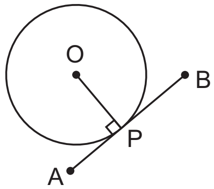
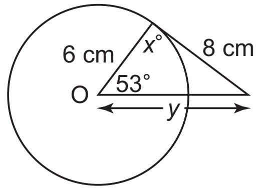
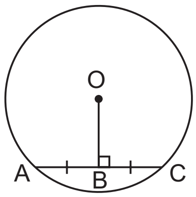
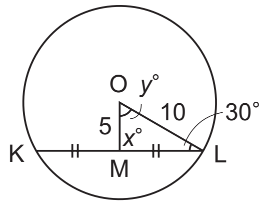
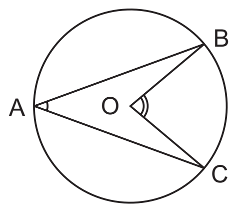
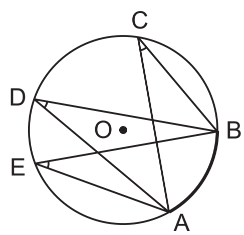
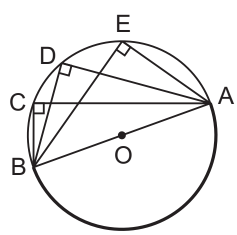
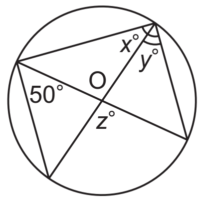
**Circle Geometry Word Search**

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R | E | S | E | M | I | C | I | R | C | L | E | E | K | P |
| A | L | C | T | S | E | S | Y | E | C | P | L | N | O | S |
| L | A | I | N | F | Y | E | O | H | L | G | O | I | M | U |
| U | U | R | E | E | R | B | O | S | N | G | N | F | M | B |
| C | Q | C | G | T | R | R | I | A | C | T | N | P | C | T |
| I | E | L | N | A | D | E | L | S | O | E | E | A | K | E |
| D | S | E | A | C | V | A | F | F | E | D | L | L | T | N |
| N | C | E | T | C | R | E | T | M | I | C | S | E | Z | D |
| E | L | C | E | T | E | A | L | A | U | T | T | I | S | E |
| P | L | A | N | R | N | Z | M | D | V | C | U | J | E | D |
| R | K | E | C | G | G | E | E | G | P | U | R | B | M | M |
| E | C | R | E | Z | T | E | E | L | G | N | A | I | R | T |
| P | A | N | M | E | R | A | D | I | U | S | O | I | C | G |
| H | C | Q | R | T | N | I | O | P | F | R | B | G | R | Q |
| Y | E | L | G | N | A | D | E | B | I | R | C | S | N | I |

Find these words in the puzzle above.
You can move in any direction to find the entire word.
A letter may be used in more than one word.

| | | |
|---|---|---|
| degrees | radius | perpendicular |
| centre | inscribed angle | bisect |
| diameter | central angle | circle |
| tangent | chord | triangle |
| circumference | point | angle |
| arc | isosceles | equal |
| point of tangency | semicircle | subtended |

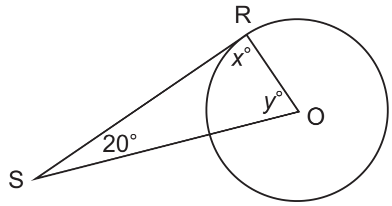
# Unit 8 Study Guide

| Skill | Description | Example |
|---|---|---|
| Recognize and apply tangent properties | <br>∠APO = ∠BPO = 90° | <br>$x° = 90°$ |
| Recognize and apply chord properties in circles | <br>If OB ⊥ AC, then AB = CB.<br>If AB = CB, then OB ⊥ AC. | <br>$x° = 90°$ and $y° = 60°$<br>$ML^2 = 10^2 - 5^2$ |
| Recognize and apply angle properties in a circle | • Inscribed and central angles<br><br>∠BOC = 2∠BAC, or<br>$\angle BAC = \frac{1}{2}\angle BOC$<br>• Inscribed angles<br><br>∠ACB = ∠ADB = ∠AEB<br>• Angles on a semicircle<br><br>∠ACB = ∠ADB = ∠AEB = 90° | <br>$x° = 90°$<br>$y° = 50°$<br>$z° = 100°$ |

# Unit 8 Review

**8.1** **1.** Find each value of $x°$ and $y°$. Segments RS and MN are tangents.

**a)**

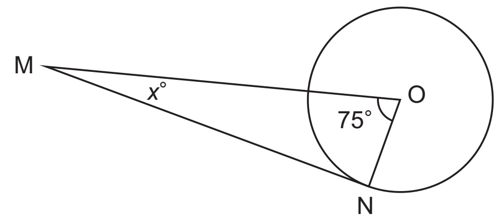



$x° =$ ______

$y° = 180° -$ ______ $-$ ______

$=$ ______

**b)**



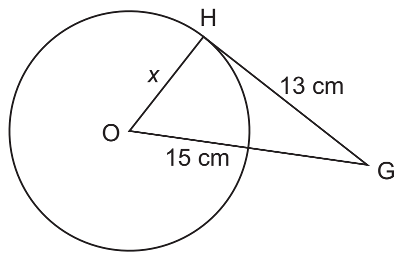

$\angle$ONM $=$ ______

$x° = 180° -$ ______ $-$ ______

$=$ ______

**2.** Find each value of $x$ to the nearest tenth. Segments GH and ST are tangents.

**a)**




$\angle$OHG $=$ ______

______ $= x^2 +$ ______

______________________

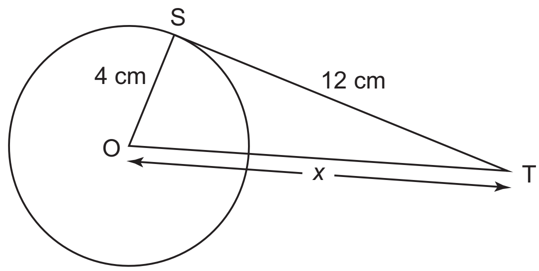
______________________

______________________

______________________

So, $x \doteq$ ________ cm

**b)**




$\angle$OST $=$ ______

______________________

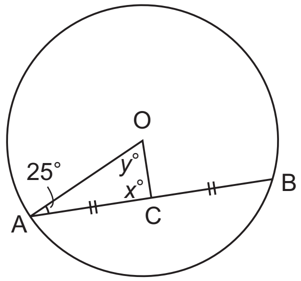
______________________

______________________

______________________

So, $x \doteq$ ________ cm

**8.2** **3.** Find the values of $x°$ and $y°$.



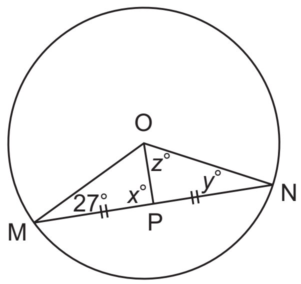

$x° =$ ______ By the chord properties

$y° =$ ______ $-$ ______ $-$ ______ By the angle sum property

______________________

$y° =$ ______

**4.** Find the values of $x°$, $y°$, and $z°$.




$x° =$ ______ By the ______________

OM = ON, so △______ is isosceles.

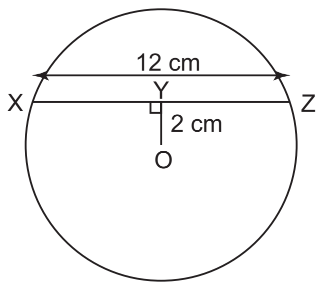
∠ONP = ∠OMP

So, $y° =$ ______

$z° =$ ______ $-$ ______ $-$ ______ ______________________

$z° =$ ______

**5.** Find the length of the radius of the circle to the nearest tenth.




$XY = \frac{1}{2} \times$ ______

$= \frac{1}{2} \times$ ______ cm

$=$ ______ cm

Draw radius OX.

$OX^2 =$ ______ $+ XY^2$

$OX^2 =$ ______ $+$ ______

______________________

______________________

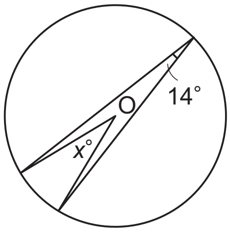
______________________

$OX \doteq$ ______

The radius is about ______ cm.

**8.3** **6.** Find each value of $x°$.

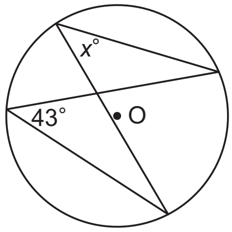
**a)**




$x° = 2 \times$ ______

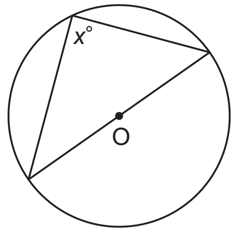
$x° =$ ______

**b)**




$x° =$ ______

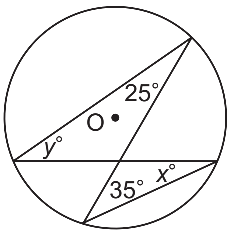
**c)**




$x° =$ ______

**7.** Find each value of $x°$ and $y°$.

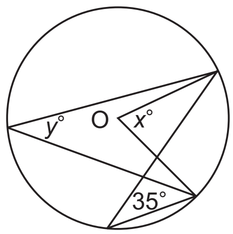
**a)**




$x° =$ ______

$y° =$ ______

**b)**

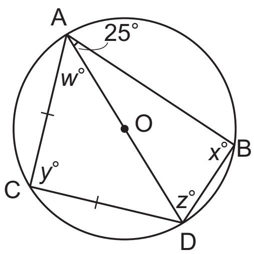



$x° = 2 \times$ ______

$=$ ______

$y° =$ ______

**8.** Find the value of $w°$, $x°$, $y°$, and $z°$.




$x° = y° =$ ______

$z° =$ ______ $-$ ______ $-$ ______ By the angle sum property

________________________

$z° =$ ______

$\triangle ACD$ is ____________. So, $\angle CDA = \angle CAD = w°$

$w° + w° =$ ______ $-$ ______ By the angle sum in $\triangle ACD$

$2w° =$ ______

$w° = \frac{\_\_\_}{2}$

$w° =$ ______