UNIT 3

Rational Numbers

Suppose you are ice fishing on Blachford Lake, NWT. The temperature at midnight is –12°C. At 6 A.M. the next day, the temperature is –11°C. What must the temperature have been at some time during the night?

What You'll Learn

- Compare and order rational numbers.
- Solve problems by adding, subtracting, multiplying, and dividing rational numbers.
- Explain and apply the order of operations with rational numbers, with and without technology.

Why It's Important

You have learned ways to use positive fractions and decimals. In some applications, such as temperature, finances, and graphing on a grid, negative fractions and negative decimals are required.

Key Words

- rational number
- irrational number

3.1 What Is a Rational Number?

FOCUS

- Compare and order rational numbers.

The label on a package of frozen cranberries says that it must be stored at a temperature between −18°C and −22°C. Name some possible temperatures. How could these temperatures be shown on a number line?

Investigate

2

➤ Determine each quotient.

$\frac{12}{2}$ $\frac{-12}{2}$ $\frac{12}{-2}$

➤ Use what you know about integer division to determine each quotient.

$\frac{11}{2}$ $\frac{-11}{2}$ $\frac{11}{-2}$

$\frac{3}{5}$ $\frac{-3}{5}$ $\frac{3}{-5}$

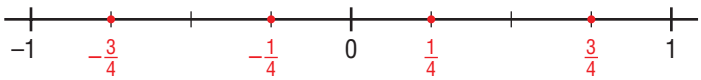
➤ On a number line, mark a point for each quotient.
How can you name the point another way?

Reflect & Share

Compare your strategies and answers with those of another pair of classmates.
Use integer division to explain what each fraction means.
How could you write each answer as a decimal?

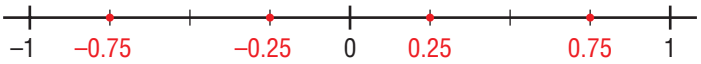
Connect

We extend a number line to the left of 0 to show negative integers.
We can also represent negative fractions on a number line.
$-\frac{3}{4}$ is the same distance to the left of 0 as $\frac{3}{4}$ is to the right of 0.



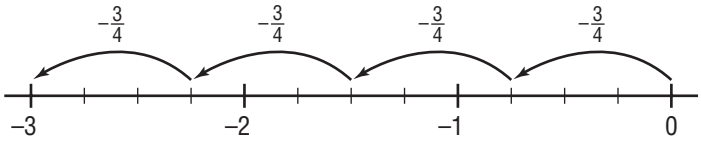
We use the same symbol to represent a negative number as we use for subtraction.

For every positive fraction, there is a corresponding negative fraction.
$-\frac{3}{4}$ and $\frac{3}{4}$ are opposites.
Any fraction can be written as a decimal; so, for every positive decimal there is a corresponding negative decimal.



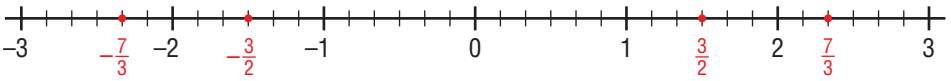
0.25 and −0.25 are opposites.

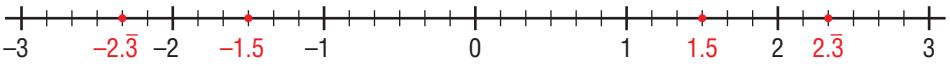
Any number that can be written as a fraction with an integer numerator and a non-zero integer denominator is a **rational number**; for example, $\frac{3}{4}, \frac{-3}{4}, \frac{3}{-4}$
To visualize $\frac{-3}{4}$, use a number line and think of $(-3) \div 4$.



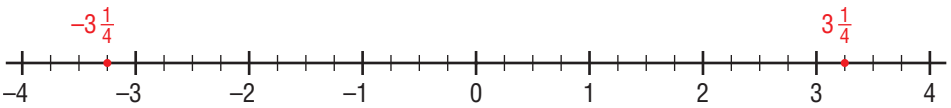
Each part is $-\frac{3}{4}$.
So, $\frac{-3}{4}$ is the same as $-\frac{3}{4}$.
The value of a fraction remains the same if its numerator and denominator are multiplied by the same non-zero number.
$\frac{3}{-4}$ can be written as $\frac{3}{-4} \times \frac{-1}{-1} = \frac{-3}{4}$
Since $\frac{3}{-4} = \frac{-3}{4}$ and $\frac{-3}{4} = -\frac{3}{4}$, then $\frac{3}{-4} = \frac{-3}{4} = -\frac{3}{4}$

A fraction can be written as a terminating or repeating decimal:





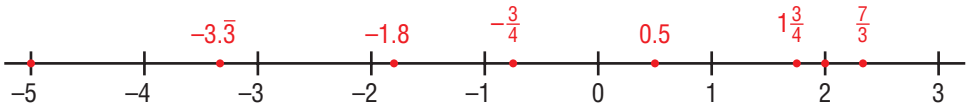
Any mixed number can be written as an improper fraction:
$3\frac{1}{4} = \frac{13}{4}$ and $-3\frac{1}{4} = -\frac{13}{4}$



So, mixed numbers are rational numbers.

Any integer can be written as a fraction with denominator 1;
for example, $-12 = \frac{-12}{1}$, so integers are rational numbers.

All these numbers are rational numbers:
$-\frac{3}{4}$, 0.5, −1.8, 0, −5, $\frac{7}{3}$, 2, $-3.\overline{3}$, $1\frac{3}{4}$



Definition of a Rational Number

A rational number is any number that can be written in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$.

Not all numbers can be written as fractions. For example, π and $\sqrt{2}$ are numbers that you have used in calculations but they cannot be written as fractions. These are **irrational numbers**.

Example 1 Writing a Rational Number between Two Given Numbers

Write 3 rational numbers between each pair of numbers.

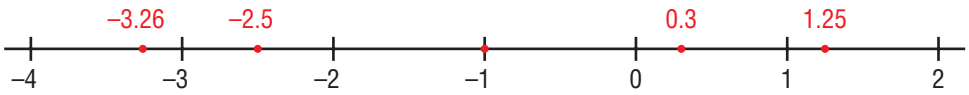
a) 1.25 and -3.26 **b)** -0.25 and -0.26 **c)** $-\frac{1}{2}$ and $\frac{1}{4}$ **d)** $-\frac{1}{2}$ and $-\frac{1}{4}$

A Solution

There are many rational numbers between any two given numbers.
Sketch or visualize a number line in each case.

a) 1.25 and -3.26

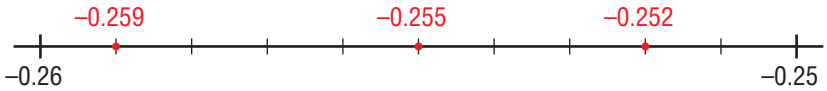
Label a number line with integers from -4 to 2.



From the number line, 3 possible rational numbers are:
-2.5, -1, and 0.3

b) -0.25 and -0.26

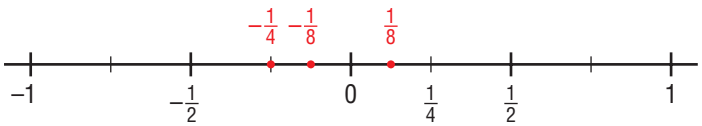
Label a number line with these rational numbers.
Divide the line into 10 equal parts.



From the number line, 3 possible rational numbers are:
-0.252, -0.255, and -0.259

c) $-\frac{1}{2}$ and $\frac{1}{4}$

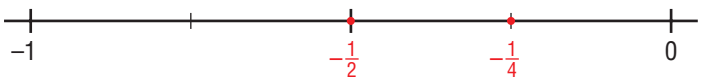
Label a number line from -1 to 1.
Divide the line into quarters.



From the number line, 3 possible rational numbers are:
$-\frac{1}{4}$, $-\frac{1}{8}$, and $\frac{1}{8}$

d) $-\frac{1}{2}$ and $-\frac{1}{4}$

Label a number line from -1 to 0. Divide the line into quarters.



Write equivalent fractions for $-\frac{1}{2}$ and $-\frac{1}{4}$ with denominators of 8 to identify fractions between the two numbers.

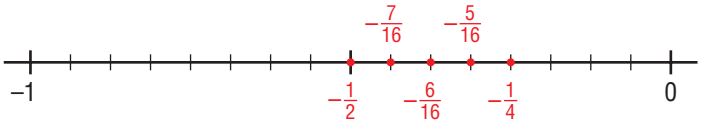
$-\frac{1}{2} = -\frac{2}{4} = -\frac{4}{8}$ $\qquad$ $-\frac{1}{4} = -\frac{2}{8}$

Between $-\frac{4}{8}$ and $-\frac{2}{8}$, there is only one fraction, $-\frac{3}{8}$, with denominator 8.

So, write equivalent fractions with denominator 16:

$-\frac{1}{2} = -\frac{2}{4} = -\frac{4}{8} = -\frac{8}{16}$ $\qquad$ $-\frac{1}{4} = -\frac{2}{8} = -\frac{4}{16}$

Divide the number line into sixteenths.



From the number line, 3 possible rational numbers are: $-\frac{5}{16}$, $-\frac{6}{16}$, and $-\frac{7}{16}$

Example 2 Ordering Rational Numbers in Decimal or Fraction Form

a) Use a number line. Order these numbers from least to greatest.

0.35, 2.5, -0.6, 1.7, -3.2, $-0.\overline{6}$

b) Order these numbers from greatest to least. Record the numbers on a number line.

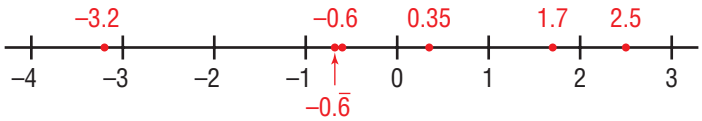
$-\frac{3}{8}, \frac{5}{9}, -\frac{10}{4}, -1\frac{1}{4}, \frac{7}{10}, \frac{8}{3}$

Solutions

a) 0.35, 2.5, -0.6, 1.7, -3.2, $-0.\overline{6}$

Mark each number on a number line.

$-0.\overline{6} = -0.666\,666\ldots$; so, $-0.\overline{6} < -0.6$

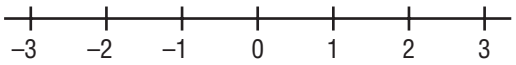


For least to greatest, read the numbers from left to right: -3.2, $-0.\overline{6}$, -0.6, 0.35, 1.7, 2.5

Method 1

b) $-\frac{3}{8}, \frac{5}{9}, -\frac{10}{4}, -1\frac{1}{4}, \frac{7}{10}, \frac{8}{3}$

Visualize a number line.



Consider the positive numbers: $\frac{5}{9}, \frac{7}{10}, \frac{8}{3}$

Only $\frac{8}{3}$ is greater than 1.

Both $\frac{5}{9}$ and $\frac{7}{10}$ are between 0 and 1.

To order $\frac{5}{9}$ and $\frac{7}{10}$, write them with a common denominator:

$9 \times 10 = 90$

$\frac{5}{9} = \frac{50}{90}$ $\quad \frac{7}{10} = \frac{63}{90}$

Since $\frac{63}{90} > \frac{50}{90}$, then $\frac{7}{10} > \frac{5}{9}$

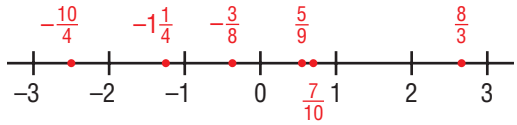
Consider the negative numbers: $-\frac{3}{8}, -\frac{10}{4}, -1\frac{1}{4}$

$-1\frac{1}{4}$ is the improper fraction $-\frac{5}{4}$, which is greater than $-\frac{10}{4}$.

$-\frac{3}{8}$ is greater than $-1\frac{1}{4}$.

From greatest to least, the numbers are:

$\frac{8}{3}, \frac{7}{10}, \frac{5}{9}, -\frac{3}{8}, -1\frac{1}{4}, -\frac{10}{4}$



Method 2

$-\frac{3}{8}, \frac{5}{9}, -\frac{10}{4}, -1\frac{1}{4}, \frac{7}{10}, \frac{8}{3}$

Write each number as a decimal.

Use a calculator when necessary.

$-\frac{3}{8} = -0.375$ $\quad \frac{5}{9} = 0.\overline{5}$

$-\frac{10}{4} = -2.5$ $\quad -1\frac{1}{4} = -1.25$

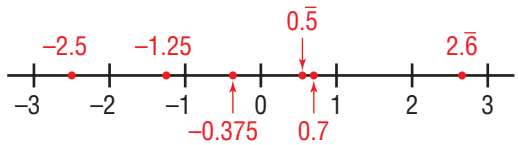
$\frac{7}{10} = 0.7$ $\quad \frac{8}{3} = 2.\overline{6}$

Mark each decimal on a number line.

Use the order of the decimals to order the fractions.

From greatest to least, the numbers are:

$\frac{8}{3}, \frac{7}{10}, \frac{5}{9}, -\frac{3}{8}, -1\frac{1}{4}, -\frac{10}{4}$

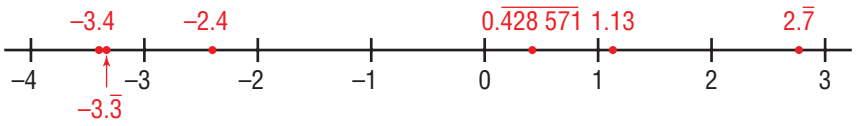


Example 3 Ordering Rational Numbers in Fraction and Decimal Form

Order these rational numbers from least to greatest.
$1.13, -\frac{10}{3}, -3.4, 2.\overline{7}, \frac{3}{7}, -2\frac{2}{5}$
Record the numbers on a number line.

A Solution

$1.13, -\frac{10}{3}, -3.4, 2.\overline{7}, \frac{3}{7}, -2\frac{2}{5}$
Write the fractions and mixed number as decimals.
$-\frac{10}{3} = -3.\overline{3}$
$\frac{3}{7} = 0.\overline{428\ 571}$
$-2\frac{2}{5} = -2.4$
Mark each decimal on a number line.



For least to greatest, read the decimals from left to right.
The order is:
$-3.4, -\frac{10}{3}, -2\frac{2}{5}, \frac{3}{7}, 1.13, 2.\overline{7}$

Discuss the ideas

1. How can you use what you know about fractions and integers to explain what a rational number is?

2. How are positive fractions and their opposites related on a number line?

3. In the definition of a rational number as $\frac{m}{n}$, where m and n are integers, why is it important that $n \neq 0$?

4. Describe the numbers that are rational, but are not positive fractions or integers.

Practice

Check

5. Identify equal rational numbers in the list that follows.

$\frac{2}{3}$ $\frac{-3}{2}$ $\frac{-2}{3}$ $-\frac{2}{3}$

$-\frac{3}{2}$ $\frac{2}{-3}$ $\frac{3}{-2}$ $\frac{3}{2}$

6. For each rational number, write two fractions that represent the same number.

a) $\frac{7}{-9}$ b) $\frac{-5}{3}$ c) $-\frac{6}{11}$

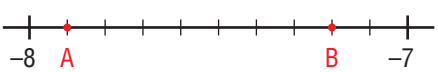
7. Write each rational number as a decimal.

a) $\frac{6}{5}$ b) $-\frac{6}{5}$

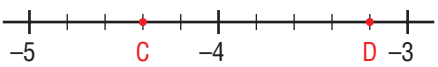
c) $\frac{9}{4}$ d) $-\frac{11}{6}$

8. Write the rational number represented by each letter on the number line, as a decimal.

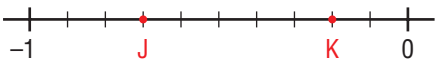
a)



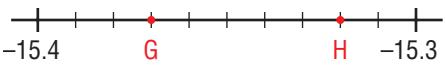
b)



c)



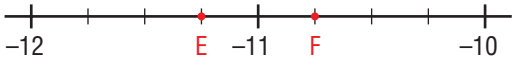
d)



9. For each pair of rational numbers in question 8, identify the greater number.

10. Write the rational number represented by each letter on the number line, as a fraction.

a)



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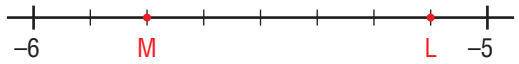

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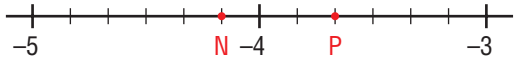
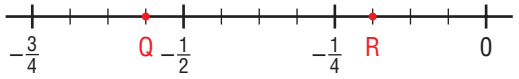


d)



11. For each pair of rational numbers in question 10, identify the lesser number.

Apply

12. Write 3 rational numbers between each pair of numbers.
Sketch a number line to show all the rational numbers.

a) 3.7, 4.2
b) −1.5, 0
c) −4.5, −4
d) −5.6, −4.5
e) −5.6, 5.7
f) 5.6, −5.7
g) −5.6, −5.7
h) −2.98, −2.99

13. The thermostat on a freezer is set at $-18°C$. The compressor on the freezer turns on and cools down the freezer when the temperature rises to $-15.5°C$. The compressor turns off when the temperature drops to $-19.5°C$.

a) Sketch a thermometer and mark the 3 freezer temperatures.

b) A package of meat must remain below $-18°C$. Should this freezer be used? Explain.

14. Write 3 rational numbers between each pair of numbers. Sketch a number line to show all the rational numbers.

a) $\frac{5}{8}, \frac{13}{8}$

b) $\frac{17}{10}, -\frac{11}{5}$

c) $-\frac{15}{4}, -\frac{11}{3}$

d) $-\frac{1}{2}, -\frac{1}{8}$

e) $\frac{1}{6}, 0.5$

f) $-0.25, -\frac{1}{3}$

g) $-\frac{14}{5}, -3$

h) $5\frac{3}{5}, 5\frac{4}{5}$

15. Sketch a number line and mark each rational number on it:
$\frac{3}{5}, -\frac{5}{7}, -\frac{8}{3}, -\frac{19}{5}$

16. Which rational number is greater? Which strategies did you use to find out?

a) 2.34, 2.3

b) $-2.34, -2.3$

c) $-1.4, 1.4$

d) $3.96, -4.12$

e) $-5.\overline{6}, -5.6$

f) $2.8\overline{6}, 2.866$

17. Which rational number is less? Explain how you know.

a) $\frac{3}{4}, \frac{3}{5}$

b) $2\frac{1}{2}, -1\frac{7}{8}$

c) $-\frac{13}{10}, -\frac{13}{5}$

d) $-\frac{11}{3}, -\frac{5}{6}$

18. Which rational number is greater? How do you know?

a) $\frac{3}{4}, \frac{6}{7}$

b) $-\frac{3}{4}, -\frac{6}{7}$

c) $-\frac{6}{7}, -\frac{7}{6}$

d) $-\frac{9}{5}, \frac{5}{9}$

19. A student said, "When I compare two numbers, I know that the lesser number is closer to 0." Is this statement always true? Sometimes true? Never true? Explain.

20. Assessment Focus

a) Mark these rational numbers on a number line:
$1.4, -\frac{11}{8}, -3.6, 4\frac{1}{3}, 0.8, -\frac{17}{3}$

b) Which rational numbers in part a are less than -1? How do you know?

c) Which rational numbers in part a are greater than -2? How do you know?

d) Write one rational number between each pair of numbers on the number line.

21. Use $<$, $>$, or $=$ to make each expression true. Justify your answers.

a) $-\frac{5}{7} \square -\frac{4}{7}$

b) $-\frac{5}{6} \square -\frac{5}{7}$

c) $-2.2 \square -\frac{11}{5}$

d) $-4.4\overline{6} \square -4.46$

22. Three hikers are returning to base camp after a mountain climbing expedition. Hiker A is 26.4 m above base camp, hiker B is 37.2 m below base camp, and hiker C is 15.7 m below base camp.

a) Represent each hiker's distance above or below base camp as a rational number.

b) Sketch and label a vertical number line to show the base camp and the positions of the hikers.

c) Which hiker is closest to base camp? Explain your reasoning.

d) Which hiker has the lowest altitude? How do you know?

23. Show each set of numbers on a number line. Order the numbers from least to greatest.

a) $1.5, -3.5, 4, 0, -2.5, 7.5$

b) $-1.7, 5.9, -3.2, -0.8, 1, 4.3$

c) $1.2, 2.1, -2.01, -1.2, 1.\overline{2}, -1.22$

d) $5.44, -5.4, -5.04, 5.\overline{4}, 5.04, -5.44$

24. Show each set of numbers on a number line. Order the numbers from greatest to least.

a) $\frac{3}{8}, -\frac{3}{4}, -\frac{1}{2}, -\frac{5}{8}, \frac{1}{4}, 0$

b) $\frac{10}{9}, -\frac{5}{3}, \frac{7}{2}, -\frac{3}{2}, -\frac{7}{6}, \frac{17}{3}$

c) $-\frac{9}{5}, -\frac{17}{10}, -1\frac{1}{2}, \frac{16}{4}, -\frac{11}{4}, \frac{21}{5}$

d) $-\frac{11}{2}, \frac{10}{3}, 2\frac{1}{4}, -\frac{8}{6}, \frac{7}{12}, -\frac{6}{4}$

25. Show each set of numbers on a number line. Order the numbers from least to greatest.

a) $3.8, \frac{3}{8}, -1.5, \frac{5}{3}, -2.3, -\frac{3}{2}$

b) $0.3, -0.\overline{3}, \frac{1}{3}, -0.3, 0.33, -3$

Take It Further

26. Use the definition of a rational number to show that each of the following numbers is rational.

a) 3 **b)** -2 **c)** -0.5 **d)** -7.45

27. Which of the following numbers do you think are rational numbers? Explain why.

a) $4.\overline{21}$ **b)** $-3.121\ 121\ 112\ 111\ 12\ldots$

c) 2.78 **d)** $-2.122\ 222\ 22\ldots$

Reflect

What is a rational number? List 3 rational numbers in decimal form and 3 rational numbers in fraction form. Show the numbers on a number line.

Start Where You Are

How Can I Learn from Others?

Three students discuss the answers to these questions:

1. Evaluate: $\frac{5}{6} + \frac{3}{4}$

2. Evaluate: $3 - 5$

1. Evaluate: $\frac{5}{6} + \frac{3}{4}$

Dan said: The sum is $\frac{8}{10}$, which simplifies to $\frac{4}{5}$.

Jesse said: Dan must be wrong; the answer has to be greater than 1.

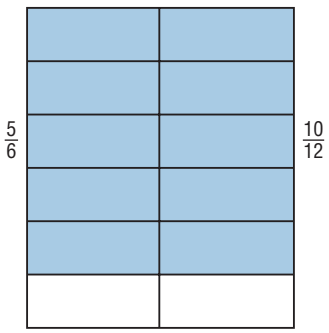
Philippe said: The answer has to be greater than $\frac{5}{6}$, and $\frac{4}{5}$ is less than $\frac{5}{6}$.

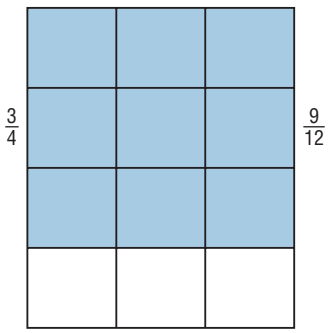
To help Dan, Jesse explained how he knew his answer was wrong:

I use benchmarks and estimate. Both $\frac{5}{6}$ and $\frac{3}{4}$ are greater than $\frac{1}{2}$, so their sum has to be greater than $\frac{1}{2} + \frac{1}{2} = 1$.

Philippe explained his strategy for adding:

I know I can add the same types of fractions. For $\frac{5}{6}$ and $\frac{3}{4}$ to be the same type, I write them as equivalent fractions with the same denominator. Then I add the numerators.





Then, $\frac{5}{6} + \frac{3}{4} = \frac{10}{12} + \frac{9}{12}$

$= \frac{10 + 9}{12}$

$= \frac{19}{12}$, or $1\frac{7}{12}$

2. Evaluate: $3 - 5$

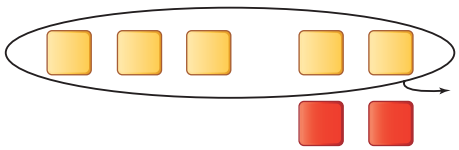
Philippe said: There is no answer because 5 is greater than 3.

Jesse said: I just switch the numbers around and calculate $5 - 3 = 2$.

Dan said: No, you can't change the order of the numbers − subtraction is not commutative. You have to think about integers.

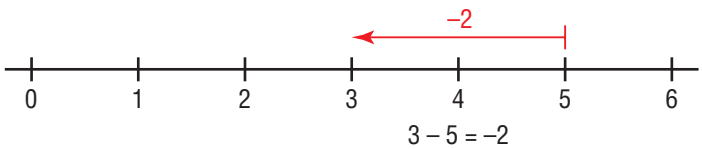
To help Philippe and Jesse, Dan explained two strategies:

➤ I can visualize coloured tiles, and add zero pairs.



$3 - 5 = -2$

➤ I can also use a number line. The difference between 2 numbers is the distance between 2 points on the number line.



$3 - 5 = -2$

Check

1. Evaluate.

a) $\frac{2}{3} + \frac{5}{2}$ **b)** $\frac{9}{8} + \frac{7}{4}$ **c)** $\frac{9}{10} + \frac{3}{5}$ **d)** $\frac{8}{3} + \frac{11}{4}$

e) $\frac{7}{2} - \frac{4}{5}$ **f)** $\frac{11}{6} - \frac{4}{3}$ **g)** $\frac{13}{4} - \frac{7}{5}$ **h)** $\frac{17}{3} - \frac{17}{6}$

2. Evaluate.

a) $7 - 3$ **b)** $3 - 7$ **c)** $-3 - 7$ **d)** $-3 - (-7)$

e) $-5 + 4$ **f)** $-6 - (-3)$ **g)** $8 - (-10)$ **h)** $-8 - 10$

3.2 Adding Rational Numbers

FOCUS

- Solve problems that require adding rational numbers.

At 6 A.M., the temperature was −3°C. By 10 A.M., the temperature had risen by 6°C. How can you use a number line to determine the temperature at 10 A.M.?

Investigate

2

Use what you know about adding integers and adding fractions to determine each sum.
Draw a number line to illustrate each sum.

$3 + 7$	$\frac{3}{8} + \frac{7}{8}$	$1\frac{3}{8} + 2\frac{7}{8}$
$-3 + 7$	$-\frac{3}{8} + \frac{7}{8}$	$-1\frac{3}{8} + 2\frac{7}{8}$
$-3 + (-7)$	$-\frac{3}{8} + \left(-\frac{7}{8}\right)$	$-1\frac{3}{8} + \left(-2\frac{7}{8}\right)$
$3 + (-7)$	$\frac{3}{8} + \left(-\frac{7}{8}\right)$	$1\frac{3}{8} + \left(-2\frac{7}{8}\right)$

Reflect & Share

Compare your strategies with those of another pair of students.
How did the first sum in each line help you determine the other sums?
How could you check your answers?
How are the strategies for adding rational numbers similar to those for adding integers and adding fractions?
Check your ideas by adding other rational numbers.

Connect

To add rational numbers in fraction form, recall how to add fractions and add integers.

➤ To add $\frac{2}{3} + \frac{1}{5}$, use a common denominator.

So, $\frac{2}{3} + \frac{1}{5} = \frac{10}{15} + \frac{3}{15}$ Write the numerators as a sum of integers.

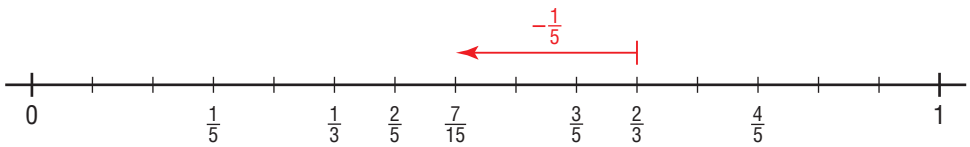
$= \frac{10 + 3}{15}$

$= \frac{13}{15}$

➤ Here are 2 strategies to add $\frac{2}{3} + \left(-\frac{1}{5}\right)$:

- Visualize a number line.

  To add $-\frac{1}{5}$, start at $\frac{2}{3}$ then move $\frac{1}{5}$ to the left.



$\frac{2}{3} + \left(-\frac{1}{5}\right) = \frac{7}{15}$

- Use equivalent fractions.

$\frac{2}{3} + \left(-\frac{1}{5}\right) = \frac{2}{3} + \left(\frac{-1}{5}\right)$

$= \frac{10}{15} + \left(\frac{-3}{15}\right)$ Add the integers in the numerator.

$= \frac{10 - 3}{15}$

$= \frac{7}{15}$

A number line is useful when the denominators of the fractions are compatible or when we want to estimate a sum.

Example 1 Adding Rational Numbers in Fraction and Mixed Number Form

Evaluate.

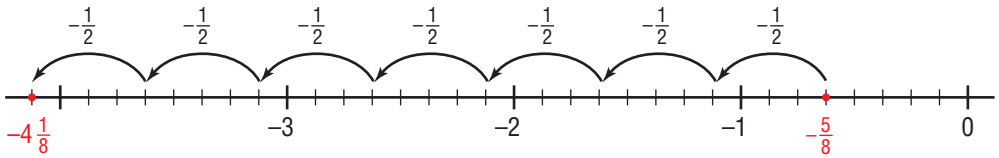
a) $-\frac{5}{8} + \left(-\frac{7}{2}\right)$ b) $-\frac{1}{4} + 2\frac{1}{6}$

A Solution

a) $-\frac{5}{8} + \left(-\frac{7}{2}\right)$

The denominators are compatible, so use a number line.

To add $-\frac{7}{2}$, start at $-\frac{5}{8}$ then move $\frac{7}{2}$ to the left.



$-\frac{5}{8} + \left(-\frac{7}{2}\right) = -4\frac{1}{8}$

b) $-\frac{1}{4} + 2\frac{1}{6}$

Write the mixed number $2\frac{1}{6}$ as the improper fraction $\frac{13}{6}$,

then write the fractions with a common denominator of 12.

Then, $-\frac{1}{4} + 2\frac{1}{6} = \frac{-3}{12} + \frac{26}{12}$

$= \frac{-3 + 26}{12}$

$= \frac{23}{12}$, or $1\frac{11}{12}$

Example 2 Adding Rational Numbers in Mixed Number Form

Evaluate.

$-3\frac{1}{3} + 2\frac{5}{6}$

Solutions

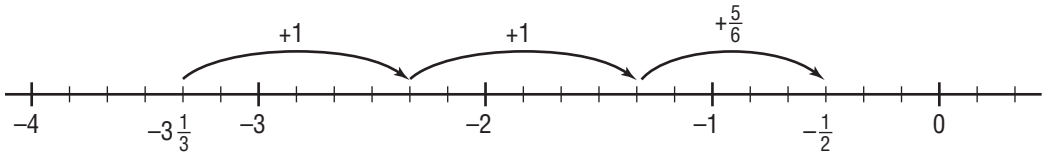
Method 1

$-3\frac{1}{3} + 2\frac{5}{6}$

Draw a number line.

Use a common denominator of 6, and divide the line into sixths.

Start at $-3\frac{1}{3}$ and move $2\frac{5}{6}$ to the right.



From the number line, $-3\frac{1}{3} + 2\frac{5}{6} = -\frac{1}{2}$

Method 2

$-3\frac{1}{3} + 2\frac{5}{6}$

Use equivalent fractions with denominator 6.

$-3\frac{1}{3} = -\frac{10}{3} = \frac{-20}{6}$

$$-3\frac{1}{3} + 2\frac{5}{6} = \frac{-20}{6} + \frac{17}{6}$$
$$= \frac{-20 + 17}{6}$$
$$= \frac{-3}{6}$$
$$= -\frac{1}{2}$$

Method 3

$-3\frac{1}{3} + 2\frac{5}{6}$

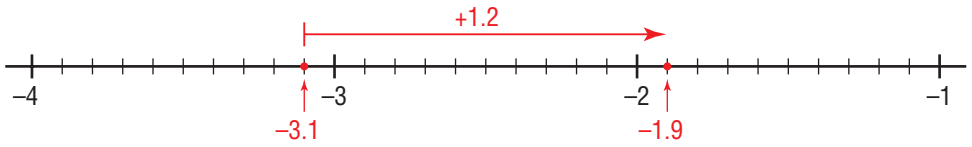
Add the whole numbers and add the fractions.

$$-3\frac{1}{3} + 2\frac{5}{6} = -3 + \left(-\frac{1}{3}\right) + 2 + \frac{5}{6}$$
$$= [-3 + 2] + \left[\left(-\frac{1}{3}\right) + \frac{5}{6}\right]$$
$$= -1 + \left(\frac{-2}{6}\right) + \frac{5}{6}$$
$$= -1 + \frac{3}{6}$$
$$= -1 + \frac{1}{2}$$
$$= -\frac{1}{2}$$

You can use what you know about adding integers and adding decimals to add rational numbers in decimal form.

Visualize a number line.

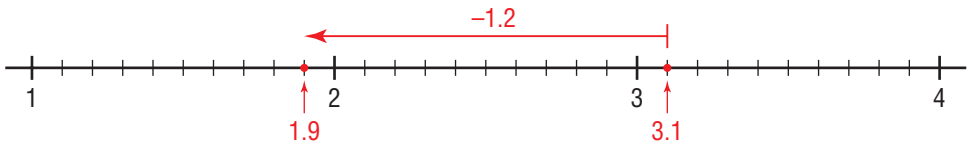
- $-3.1 + 1.2$



$-3.1 + 1.2 = -1.9$ This is an addition statement.

- $3.1 + (-1.2)$

  Remember that when we add a negative number, we move to the left.



$3.1 + (-1.2) = 1.9$

Example 3 Solving a Problem by Adding Rational Numbers

At the beginning of June, the Frosty Snow Blower Company was $235.46 in debt. By the end of August, the company had increased its debt by $156.71.

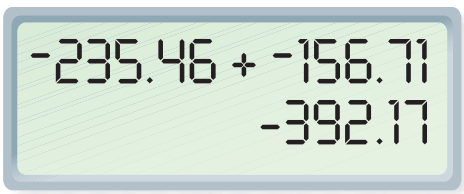
a) Use a rational number to represent each amount.

b) Calculate how much the company owed at the end of August.

A Solution

a) A debt of $235.46 can be represented as -235.46.
A debt of $156.71 can be represented as -156.71.

b) At the end of August, the company owed:
$-235.46 + (-156.71)$
Use a calculator.



$-235.46 + (-156.71) = -392.17$
At the end of August, the company owed $392.17.

Discuss the ideas

1. How can you use what you know about representing the sum of 2 integers on a number line to add 2 rational numbers?

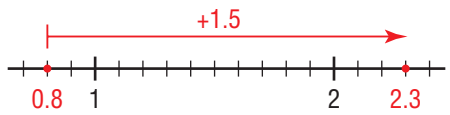
2. How can you use what you know about adding integers and adding fractions to add 2 rational numbers in fraction form?

Practice

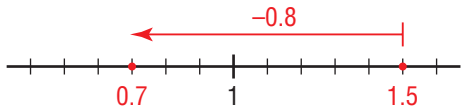
Check

3. Write the addition statement that each number line represents.

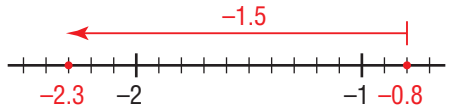
a)



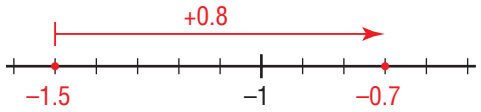
b)



c)

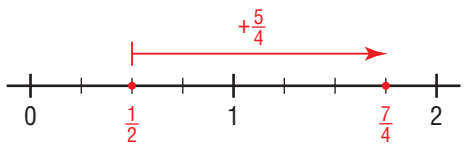


d)

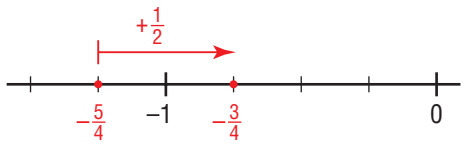


4. Write the addition statement that each number line represents.

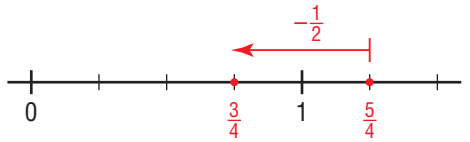
a)



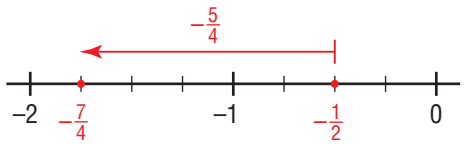
b)



c)



d)



5. Determine each sum.

a) i) $3 + 2$ ii) $3.8 + 2.4$

b) i) $-3 + (-2)$ ii) $-3.8 + (-2.4)$

c) i) $-3 + 2$ ii) $-3.8 + 2.4$

d) i) $3 + (-2)$ ii) $3.8 + (-2.4)$

6. Which of the following expressions have the same sum as $-2.3 + (-1.9)$? Use a number line to explain how you know.

a) $2.3 + 1.9$

b) $(-2.3) + 1.9$

c) $-1.9 + (-2.3)$

d) $(-2.3) + (-1.9)$

7. Determine each sum.

a) i) $9 + 3$ ii) $\frac{9}{2} + \frac{3}{2}$

b) i) $-9 + (-3)$ ii) $-\frac{9}{2} + \left(-\frac{3}{2}\right)$

c) i) $-9 + 3$ ii) $-\frac{9}{2} + \frac{3}{2}$

d) i) $9 + (-3)$ ii) $\frac{9}{2} + \left(-\frac{3}{2}\right)$

8. Which of the following expressions have the same sum as $-\frac{3}{4} + \frac{7}{8}$? Use a number line to show how you know.

a) $-\frac{3}{4} + \left(-\frac{7}{8}\right)$ b) $-\frac{7}{8} + \frac{3}{4}$

c) $\frac{7}{8} + \left(-\frac{3}{4}\right)$ d) $\frac{7}{8} + \frac{3}{4}$

Apply

9. Use integers to estimate each sum. Then, determine each sum.

a) $-5.6 + 3.2$

b) $7.95 + (-4.51)$

c) $-0.325 + (-32.5)$

d) $-123.5 + 27.45$

e) $82.001 + 100.28$

f) $-17.84 + (-0.098)$

10. Is it possible to add 2 rational numbers and get a sum that is less than both the numbers you added? Explain your reasoning.

11. Determine each sum.

a) $-\frac{2}{3} + \frac{1}{2}$ b) $\frac{4}{5} + \left(-\frac{1}{3}\right)$

c) $-\frac{11}{4} + \left(-\frac{6}{5}\right)$ d) $\frac{13}{5} + \frac{9}{2}$

e) $-2\frac{1}{3} + \left(-1\frac{3}{4}\right)$ f) $\frac{9}{5} + \left(-\frac{17}{6}\right)$

g) $-3\frac{3}{4} + 4\frac{5}{8}$ h) $1\frac{5}{6} + \left(-5\frac{2}{3}\right)$

i) $-3\frac{1}{4} + \left(-2\frac{1}{6}\right)$ j) $2\frac{3}{5} + \left(-1\frac{7}{8}\right)$

12. **Assessment Focus** What can you say about the sign of the sum of 2 rational numbers in each case? Include examples and explain your reasoning.

a) Both rational numbers are positive.

b) Both rational numbers are negative.

c) One rational number is positive and one rational number is negative.

13. Zoe owes her mother $36.25, then Zoe borrows another $25.35.

a) Write each amount as a rational number.

b) Use the numbers in part a.

i) Write an expression for the amount Zoe owes.

ii) How much does Zoe owe?

c) Zoe pays back $14.75.

i) Write an expression for the amount Zoe now owes.

ii) How much does Zoe now owe?

14. Estimate whether each sum is greater than or less than 0. Explain how you know. Calculate to check your prediction.

a) $-0.61 + 0.23$ b) $12.94 + (-12.56)$

c) $-\frac{7}{3} + \left(\frac{17}{5}\right)$ d) $\frac{7}{4} + \left(-\frac{6}{5}\right)$

15. On Tuesday, December 23rd, the lowest temperature in Winnipeg was -13.4°C. By noon the next day, the temperature had increased by 5.7°C.

a) What was the temperature at noon?

b) On Wednesday, December 24th, the lowest temperature was 3.7°C less than the lowest the previous day. What was the lowest temperature on Wednesday?

c) Sketch a thermometer to show these changes in temperature.

16. For each pair of expressions below, how can you tell which sum is greater without adding? Explain your reasoning. Determine each sum to check.

a) i) $-9.23 + 3.46$ ii) $9.23 + (-3.46)$

b) i) $-\frac{2}{3} + \left(-\frac{3}{4}\right)$ ii) $-\frac{2}{3} + \frac{3}{4}$

17. In January, Keith earned $45.50 babysitting and $22.25 shovelling snow. He spent $15.77 on a CD, and $33.10 on a computer game.

a) Write each amount above as a rational number. Justify your choice of sign for each number.

b) Write an addition statement that represents Keith's balance at the end of January.

c) What is Keith's balance?

18. The table shows the money earned and spent by Lucille in the first six months of running her new business, Lucille's Café.

Item	Income	Expense
New tables and chairs		$545.50
New stove		$978.44
Profit on food	$2115.70	
Repair of roof		$888.00
Profit on coffee	$2570.40	
Salary of part-time cook		$2540.20

Did Lucille's business make a profit in the first six months? Use rational numbers in your explanation.

19. Use a calculator to help determine a rational number that makes each sentence true.

a) $5.6 + \square \leq 9.1$

b) $11.8 + (-\square) \leq 23.4$

c) $-7.2 + \square \geq 7.2$

d) $-7.2 + \square \leq 7.2$

Take It Further

20. Determine the missing rational number in each addition statement. What strategies did you use?

a) $-\frac{3}{4} + \square = \frac{7}{8}$

b) $\square + \frac{4}{5} = -\frac{2}{3}$

c) $\square + \left(-\frac{5}{2}\right) = 3\frac{1}{8}$

d) $\frac{7}{3} + \square = -\frac{5}{4}$

21. Determine the range of numbers that makes this sentence true. Explain your reasoning.

$7.9 + \square \leq 11.2$

22. Use any four of the rational numbers:

$-1, -2, -3, -4, 1, 2, 3, 4$

in the boxes below to make an expression with the greatest sum less than 0. Explain how you know you have determined the greatest sum less than 0.

$$\frac{\square}{\square} + \frac{\square}{\square}$$

Reflect

Before you add 2 rational numbers, how can you tell if their sum will be positive or negative? Include both fraction and decimal examples in your explanation.

3.3 Subtracting Rational Numbers

FOCUS

- Solve problems that require subtracting rational numbers.

Canada's national debt was $559 billion in 1999.
By 2008, this debt had been reduced to $467 billion.
How would you write each amount as a rational number?
How could you use a number line to calculate the difference in debt?

Investigate

2

Here is part of a stock market report from February 5, 2008, for some Canadian companies.

Company	Stock price at the end of the day ($)	Stock price at the start of the day ($)
Bombardier	4.670	4.710
Canadian National Railway	50.630	51.330
Canadian Tire Corporation	64.840	65.970
Potash Corporation of Saskatchewan	144.580	144.15

For each stock:

- Determine: (price at the end of the day) − (price at the start of the day)
- What does it mean when this difference in prices is positive? Is negative?
- Sketch a number line to show each subtraction.
- Use rational numbers to write a subtraction statement.

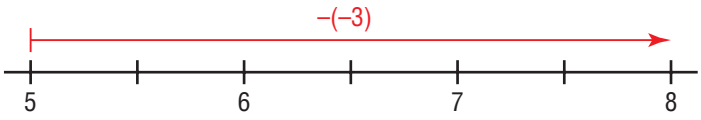
Reflect & Share

Compare your strategies and answers with those of another pair of students.
How did you use what you know about subtracting integers to subtract rational numbers?
How could you check your answers?

Connect

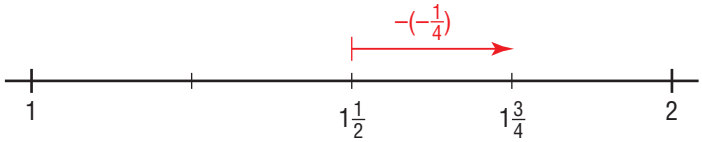
To subtract 2 rational numbers, we use a strategy similar to that for subtracting integers.

- For $5 - (-3)$, add the opposite: $5 + (+3)$
  Start at 5 then move 3 to the right.



$5 - (-3) = 8$

- For $1\frac{1}{2} - \left(-\frac{1}{4}\right)$, add the opposite: $1\frac{1}{2} + \left(+\frac{1}{4}\right)$
  Start at $1\frac{1}{2}$ then move $\frac{1}{4}$ to the right.



$1\frac{1}{2} - \left(-\frac{1}{4}\right) = 1\frac{3}{4}$

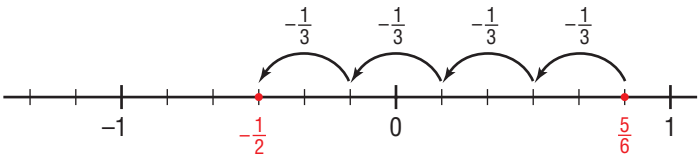
Example 1 Subtracting Rational Numbers in Fraction and Mixed Number Form

a) $\frac{5}{6} - \frac{4}{3}$ b) $-\frac{5}{4} - \left(-3\frac{1}{5}\right)$

A Solution

a) $\frac{5}{6} - \frac{4}{3}$

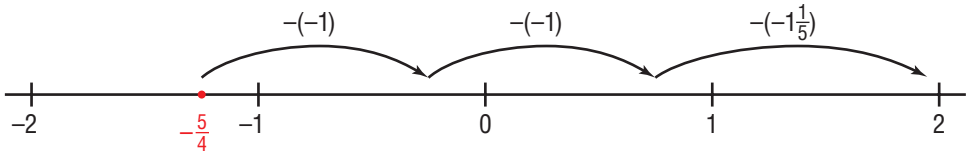
On a number line, start at $\frac{5}{6}$ then move $\frac{4}{3}$ to the left.



$\frac{5}{6} - \frac{4}{3} = -\frac{1}{2}$

b) $-\frac{5}{4} - \left(-3\frac{1}{5}\right)$

Visualize a number line to estimate the difference.



The difference is a little less than 2.

Use equivalent fractions to calculate the difference.

Write $-3\frac{1}{5}$ as the improper fraction $-\frac{16}{5}$.

So, $-\frac{5}{4} - \left(-3\frac{1}{5}\right) = -\frac{5}{4} - \left(-\frac{16}{5}\right)$

$= -\frac{25}{20} - \left(-\frac{64}{20}\right)$

$= \frac{-25 - (-64)}{20}$

$= \frac{-25 + 64}{20}$

$= \frac{39}{20}$, or $1\frac{19}{20}$

To subtract the integers in the numerator, add the opposite.

Example 2 Subtracting Rational Numbers in Mixed Number Form

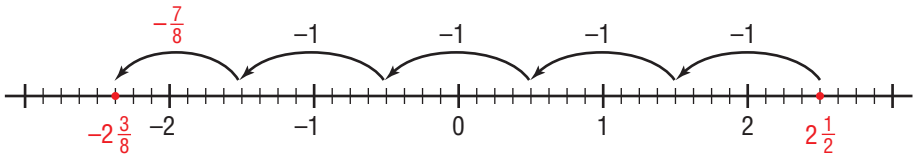
Evaluate. $2\frac{1}{2} - 4\frac{7}{8}$

Solutions

Method 1

$2\frac{1}{2} - 4\frac{7}{8}$

On a number line, start at $2\frac{1}{2}$ then move $4\frac{7}{8}$ to the left.



$2\frac{1}{2} - 4\frac{7}{8} = -2\frac{3}{8}$

Method 2

$2\frac{1}{2} - 4\frac{7}{8}$

Write each mixed number as an improper fraction.

$2\frac{1}{2} - 4\frac{7}{8} = \frac{5}{2} - \frac{39}{8}$ Use equivalent fractions.

$= \frac{20}{8} - \frac{39}{8}$

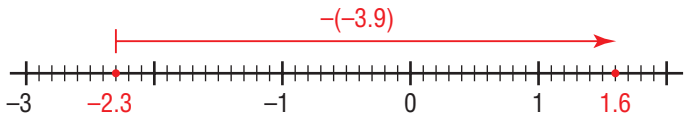
$= \frac{20 - 39}{8}$

$= \frac{-19}{8}$

$= -\frac{19}{8}$, or $-2\frac{3}{8}$

We can use a number line to subtract rational numbers in decimal form.

To subtract $-2.3 - (-3.9)$, add the opposite: $-2.3 + (+3.9)$



$-2.3 - (-3.9) = -2.3 + (+3.9)$

$= -2.3 + 3.9$

$= 1.6$

Example 3 Solving a Problem by Subtracting Rational Numbers

A diver jumps off a cliff that is 14.7 m above sea level. After hitting the water, he plunges 3.8 m below the surface of the water before returning to the surface.

a) Use rational numbers to represent the difference in heights from the top of the cliff to the bottom of his dive. Sketch a number line.

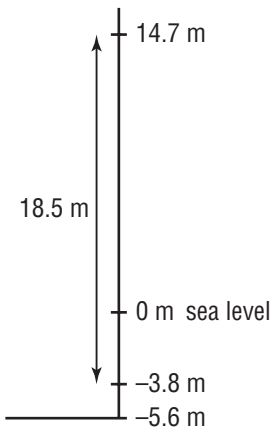
b) The water is 5.6 m deep. What is the distance from the ocean floor to the bottom of the dive?

A Solution

a) A distance measured above the water can be considered as positive.
A distance measured below the water can be considered as negative.
The diver travels 14.7 m above the water and -3.8 m below the water.
The difference in heights is: $14.7 - (-3.8)$
From the number line:

$14.7 - (-3.8) = 14.7 + 3.8$ Adding the opposite decimal
$= 18.5$

The diver travelled 18.5 m.



b) The diver travelled -3.8 m below the surface.
The ocean floor is -5.6 m below the surface.
The difference in heights is: $-5.6 - (-3.8)$

$-5.6 - (-3.8) = -5.6 + 3.8$
$= -1.8$

The distance from the bottom of the ocean floor to the bottom of the dive is 1.8 m.

Discuss the ideas

1. When you use a number line to subtract 2 rational numbers, how do you know in which direction to move?

2. How can you use what you know about subtracting integers and subtracting fractions to subtract 2 rational numbers in fraction form?

Practice

Check

3. Determine each difference.

a) i) $5 - 3$ ii) $5.1 - 3.3$

b) i) $-5 - 3$ ii) $-5.1 - 3.3$

c) i) $-3 - (-5)$ ii) $-3.3 - (-5.1)$

d) i) $3 - 5$ ii) $3.3 - 5.1$

4. Which of the following expressions have the same answer as $-7.2 - 1.8$? How do you know?

a) $7.2 - 1.8$ **b)** $-7.2 - (-1.8)$

c) $1.8 - (-7.2)$ **d)** $-1.8 - 7.2$

5. Determine each difference.

a) i) $11 - 2$ ii) $\frac{11}{5} - \frac{2}{5}$

b) i) $-11 - 2$ ii) $-\frac{11}{5} - \frac{2}{5}$

c) i) $11 - (-2)$ ii) $\frac{11}{5} - \left(-\frac{2}{5}\right)$

d) i) $2 - (-11)$ ii) $\frac{2}{5} - \left(-\frac{11}{5}\right)$

Apply

6. Which of the following expressions have the same answer as $-\frac{3}{10} - \frac{9}{5}$? How do you know?

a) $-\frac{3}{10} - \left(-\frac{9}{5}\right)$ **b)** $\frac{3}{10} - \frac{9}{5}$

c) $-\frac{9}{5} - \frac{3}{10}$ **d)** $\frac{9}{5} - \frac{3}{10}$

7. Use integers to estimate each difference. Then, determine each difference.

a) $10.8 - 3.5$ **b)** $-37.23 - 48.54$

c) $50.06 - (-14.67)$ **d)** $64.19 - 95.76$

e) $-28.31 - 9.72$ **f)** $70.59 - (-81.25)$

8. On January 25th, 2008, the lowest temperature in Iqaluit, Nunavut, was $-28.5°C$.
On the same day, the lowest temperature in Inuvik, Northwest Territories, was $-33.1°C$.

a) What is the difference in these temperatures?

b) Why are there two possible answers to part a?

9. Determine each difference.

a) $\frac{17}{3} - \frac{19}{2}$ **b)** $-\frac{13}{5} - \frac{7}{3}$ **c)** $1\frac{5}{6} - 6\frac{3}{4}$

d) $-\frac{19}{6} - \frac{7}{8}$ **e)** $\frac{15}{4} - \frac{5}{12}$ **f)** $-2\frac{1}{8} - \left(-4\frac{1}{3}\right)$

10. **Assessment Focus** Is it possible to subtract 2 rational numbers and get a difference that is greater than both the numbers you subtracted? Explain your reasoning. Include examples of rational numbers in decimal form and in fraction form.

11. In Asia, the lowest point on land is the shore of the Dead Sea, which is 417.5 m below sea level. The highest point is the peak of Mount Everest, which 8844.43 m above sea level.

a) Write each measurement above as a rational number.

b) Write a subtraction statement that represents the distance between the highest point and the lowest point. What is this distance?

12. Predict whether each answer is positive or negative. Explain how you know. Calculate to check your prediction.

a) $-3.86 - 41.12$ **b)** $1.32 - (-5.79)$

c) $-\frac{5}{4} - \left(-\frac{7}{2}\right)$ **d)** $-\frac{23}{5} - \frac{5}{3}$

13. Evaluate each expression.

a) $\frac{3}{5} - \left(-\frac{1}{2}\right) + \frac{2}{3}$

b) $-2.34 + 8.6 + (-5.71)$

c) $-\frac{16}{5} - \left(-\frac{14}{3}\right) + \frac{13}{4}$

d) $23.5 + (-12.61) - 3.2$

14. Determine a rational number that makes each statement true. Use a calculator to check your answer.

a) $-1.2 - \square \leq 3.7$

b) $4.3 - \square \geq 8.9$

c) $\square - 2.9 \geq 5.3$

d) $\square - 7.2 \leq -10.9$

Take It Further

15. Determine the missing number in each subtraction statement.

a) $\square - 28.4 = 37.3$ **b)** $\frac{9}{10} - \square = \frac{3}{5}$

c) $\square - 0.05 = -2.08$ **d)** $\frac{11}{6} - \square = -\frac{7}{3}$

e) $-1.25 - \square = 3.75$ **f)** $-3\frac{1}{2} - \square = 5\frac{1}{4}$

16. Find two pairs of rational numbers that make each equation true.

a) $-7.4 + \square - \square = -10.9$

b) $\square - (-12.8) + \square = -1.1$

c) $-21.6 - \square - \square = -15.4$

17. Determine the range of numbers that makes each sentence true. Explain your thinking.

a) $-11.8 - \square \leq 5.7$

b) $6.3 - \square \geq 9.4$

Reflect

How is subtracting 2 rational numbers similar to adding 2 rational numbers? How is it different? Include examples of rational numbers in your explanation.

Mid-Unit Review

3.1

1. a) Sketch a number line. On the line, place each rational number below.
$-1.3, 2\frac{3}{4}, 1.51, -\frac{8}{5}, -\frac{9}{3}$

b) Which numbers in part a are less than -1.5? Explain how you know.

2. Order the following rational numbers from least to greatest. Place each number on a number line to support your answer.
$-\frac{6}{5}, 1.2, -1.1, -\frac{1}{4}, 0.2, -1\frac{3}{8}$

3. Replace each $\square$ with $<$ or $>$.
How could you check your answers?

a) $-\frac{2}{3} \square -\frac{3}{4}$ b) $-\frac{8}{3} \square -\frac{9}{4}$

c) $-2.5 \square 0.5$ d) $-\frac{4}{5} \square -0.9$

4. Identify a rational number between each pair of numbers. Sketch a number line to illustrate each answer.

a) 1.2, 1.4 b) $-\frac{3}{4}, \frac{5}{8}$

c) $0.4, \frac{1}{3}$ d) $-1.05, -\frac{9}{10}$

3.2

5. a) How can you determine the sign of the sum of two numbers before you add them?

b) Determine the sign of each sum, then check by using a calculator.

i) $2.35 + 3.47$

ii) $-5.783 + (-0.247)$

iii) $-\frac{2}{3} + \left(-1\frac{1}{8}\right)$

iv) $-5.27 + 6.58$

v) $-\frac{17}{5} + \frac{4}{9}$

vi) $0.085 + (-0.125)$

6. Determine each sum.

a) $8.37 + 0.58$ b) $-21.25 + (-36.57)$

c) $-157.4 + 32.7$ d) $\frac{5}{8} + \left(-\frac{1}{9}\right)$

e) $-8\frac{1}{4} + 5\frac{1}{5}$ f) $-\frac{5}{3} + \left(-\frac{23}{7}\right)$

3.3

7. The temperature of a freezer changed from $-16.1°C$ to $-14.7°C$.

a) i) By how much did the temperature change?

ii) Is this an increase or a decrease in temperature? Explain how you know.

b) By how much does the temperature need to change again before it is at $-3.8°C$?

8. Determine each difference.

a) $40.25 - 63.10$ b) $-112.2 - (-14.8)$

c) $\frac{2}{5} - \frac{9}{10}$ d) $-4\frac{4}{9} - 3\frac{5}{6}$

e) $-1.8 - 4.3$ f) $\frac{23}{8} - \left(-\frac{7}{2}\right)$

9. The lowest point on land in North America is Death Valley at 86 m below sea level. The highest point is the peak of Mt. McKinley at 6193.7 m above sea level. How can you use rational numbers to calculate the distance between these two points?

10. a) How can you determine the sign of the difference of two numbers before you subtract them?

b) Determine the sign of each difference, then check by using a calculator.

i) $62.4 - 53.7$ ii) $-0.54 - 1.98$

iii) $\frac{1}{12} - \frac{9}{10}$ iv) $5\frac{2}{3} - \left(-7\frac{1}{2}\right)$

Closest to Zero

GAME

How to Play

An Ace is worth 1, a Jack is worth 11, a Queen is worth 12, and a King is worth 13.
All the red cards are negative and the black cards are positive.

1. For each round, each player is dealt 4 cards.
2. Each player organizes her 4 cards to create 2 proper fractions – two cards are the numerators and two are the denominators.
3. Each player chooses to add or subtract her 2 fractions. This is then the value of that player's hand.
4. The winner of the round is the person whose hand has a value closest to 0. The winner gets 1 point.
   If a player has a hand whose value *is* 0, then that person wins the round and gets 2 points.
   Players record their points.
5. The cards are shuffled and play continues with the next round.
   The first player to get 10 points is the winner.

Play the game a few times.
What strategies do you have for winning a round?

You will need

- a deck of 52 playing cards
- a calculator (optional)

Number of Players

- 2 to 4

Goal of the Game

- To add or subtract fractions to get an answer that is close to 0

3.4 Multiplying Rational Numbers

FOCUS

- Solve problems that require multiplying rational numbers.

What strategies do you use:

- to multiply two integers such as $(-9) \times 8$?
- to multiply two fractions such as $\frac{3}{4} \times \frac{5}{2}$?

Investigate

2

➤ Use what you know about multiplying integers and multiplying fractions to predict each product of rational numbers.

6×8	5×7	$\frac{6}{5} \times \frac{8}{7}$		
$(-7) \times 9$	4×2	$\left(-\frac{7}{4}\right) \times \frac{9}{2}$	$\left(\frac{-7}{4}\right) \times \frac{9}{2}$	$\left(\frac{7}{-4}\right) \times \frac{9}{2}$
$(-8) \times (-6)$	3×5	$\left(-\frac{8}{3}\right) \times \left(-\frac{6}{5}\right)$	$\left(\frac{-8}{3}\right) \times \left(\frac{-6}{5}\right)$	$\left(\frac{8}{-3}\right) \times \left(\frac{6}{-5}\right)$
$9 \times (-3)$	2×10	$\frac{9}{2} \times \left(-\frac{3}{10}\right)$	$\frac{9}{2} \times \left(\frac{-3}{10}\right)$	$\frac{9}{2} \times \left(\frac{3}{-10}\right)$

➤ Use a calculator to check your predictions.

➤ Choose 2 different rational numbers in fraction form. Determine their product. Check with a calculator.

Reflect & Share

Share your answers with another pair of students. Explain to each other how you found the products. How did the first 2 products in each line help you determine the next products?

Connect

To multiply 2 rational numbers, use the properties for determining the sign of the product of 2 integers to predict the sign of the product of the rational numbers. Then:

➤ If the rational numbers are in fraction form:
Use the procedures for multiplying 2 fractions to determine the numerical value of the product.

➤ If the rational numbers are in decimal form:
Use the procedures for multiplying 2 decimals to determine the numerical value of the product.

For example,

- When two rational numbers have the same sign, their product is positive.
  $\left(-\frac{3}{2}\right) \times \left(-\frac{1}{5}\right) = \frac{3}{10}$ and $\frac{3}{2} \times \frac{1}{5} = \frac{3}{10}$
  $(-1.5) \times (-1.8) = 2.7$ and $1.5 \times 1.8 = 2.7$
- When two rational numbers have opposite signs, their product is negative.
  $\frac{3}{2} \times \left(-\frac{1}{5}\right) = -\frac{3}{10}$ and $\left(-\frac{3}{2}\right) \times \frac{1}{5} = -\frac{3}{10}$
  $(-1.5) \times 1.8 = -2.7$ and $1.5 \times (-1.8) = -2.7$

When we use brackets to write a product statement, we do not need the multiplication sign. For the rational numbers above, we can write $\frac{3}{2} \times \left(-\frac{1}{5}\right)$ as $\left(\frac{3}{2}\right)\left(-\frac{1}{5}\right)$, and $(-1.5) \times 1.8$ as $(-1.5)(1.8)$.

Example 1 Multiplying Rational Numbers in Fraction or Mixed Number Form

Determine each product.

a) $\left(-\frac{11}{7}\right)\left(-\frac{21}{44}\right)$ b) $\left(2\frac{2}{3}\right)\left(-1\frac{3}{4}\right)$

A Solution

a) $\left(-\frac{11}{7}\right)\left(-\frac{21}{44}\right)$

Predict the sign of the product: since the fractions have the same sign, their product is positive.

Simplify the fractions before multiplying.

$$\left(-\frac{11}{7}\right)\left(-\frac{21}{44}\right) = \left(-\frac{\overset{1}{\cancel{11}}}{\underset{1}{\cancel{7}}}\right)\left(-\frac{\overset{3}{\cancel{21}}}{\underset{4}{\cancel{44}}}\right)$$

$$= \frac{1 \times 3}{1 \times 4}$$

$$= \frac{3}{4}$$

So, $\left(-\frac{11}{7}\right)\left(-\frac{21}{44}\right) = \frac{3}{4}$

Look for common factors in the numerators and denominators:
11 and 44 have the common factor 11.
21 and 7 have the common factor 7.
Divide numerator and denominator by their common factors.
Then multiply the numerators and multiply the denominators.

b) $\left(2\frac{2}{3}\right)\left(-1\frac{3}{4}\right)$

Since the fractions have opposite signs, their product is negative.

Write the mixed numbers as fractions.

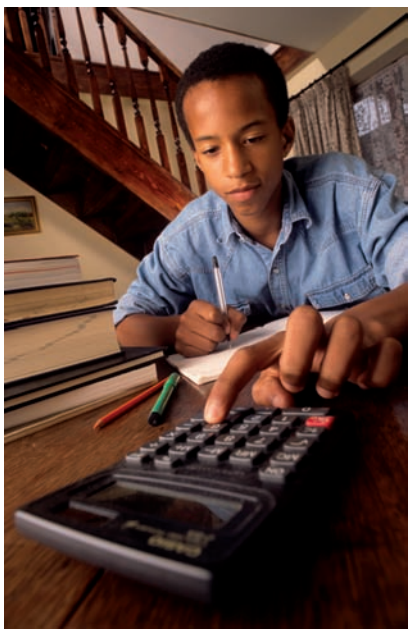
$$\left(\frac{8}{3}\right)\left(-\frac{7}{4}\right) = \left(\frac{\overset{2}{\cancel{8}}}{3}\right)\left(-\frac{7}{\underset{1}{\cancel{4}}}\right)$$ Dividing numerator and denominator by their common factor 4

$$= \frac{(2)(-7)}{(3)(1)}$$

$$= -\frac{14}{3}$$

$$= -4\frac{2}{3}$$

So, $\left(2\frac{2}{3}\right)\left(-1\frac{3}{4}\right) = -4\frac{2}{3}$



Example 2 Multiplying Rational Numbers to Solve Problems

On February 5, 2008, the price of a share in CIBC changed by −$1.640.
A person owns 35 shares. By how much did those shares change in value that day?

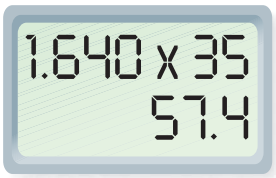
Solutions

Method 1

Change in value:
$-\$1.640 \times 35$
Since the rational numbers have opposite signs, their product is negative.
To determine the product: $(-1.64)(35)$, multiply integers, then estimate to place the decimal point.
$(-164)(35) = -5740$
Estimate to place the decimal point:
Since -1.64 is close to -2, then
$(-1.64)(35)$ is close to $(-2)(35) = -70$.
So, place the decimal point after the 7 in -5740.
$-\$1.640 \times 35 = -\$57.40$
The shares lost $57.40 that day.

Method 2

Change in value:
$-\$1.640 \times 35$
Use a calculator.
Key in 1.640×35 to display: 57.4



Since the rational numbers have opposite signs, their product is negative, so we do not need to enter the negative sign.

$-\$1.640 \times 35 = -\$57.40$
The shares lost $57.40 that day.

Example 3 Multiplying Rational Numbers in Decimal Form

Determine each product.

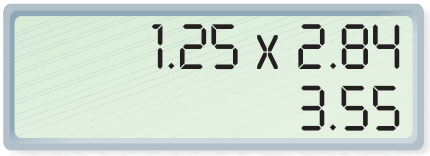
a) $(0.8)(-2.4)$ **b)** $(-1.25)(-2.84)$

A Solution

a) $(0.8)(-2.4)$
Since the rational numbers have opposite signs, their product is negative.
Use mental math to determine the product:
$(8)(-24) = -192$
Estimate to place the decimal point:
Since 0.8 is close to 1 and -2.4 is close to -2, then
$(0.8)(-2.4)$ is close to $(1)(-2) = -2$.
So, place the decimal point after the 1 in -192.
Then, $(0.8)(-2.4) = -1.92$

b) $(-1.25)(-2.84)$

When there are more than 2 digits in both numbers being multiplied, use a calculator.
The rational numbers have the same sign, so their product is positive.
Key in 1.25×2.84 to display: 3.55



$(-1.25)(-2.84) = 3.55$

Discuss the ideas

1. Why does it help to predict the sign of a product before you multiply 2 rational numbers?

2. Why does it make sense that the rules for signs when you multiply integers must apply when you multiply rational numbers?

Practice

Check

3. Predict which products are greater than 0, then multiply to determine each product. Explain the strategy you used to predict.
   a) $3 \times (-5.2)$
   b) $2.6 \times (-4)$
   c) $(-1.3) \times 5$
   d) $(-0.9) \times (-7.1)$

4. Predict which products are less than 0, then multiply to determine each product. Explain the strategy you used to predict.
   a) $(-3) \times \frac{2}{3}$
   b) $\left(-\frac{1}{4}\right) \times (-5)$
   c) $\left(\frac{4}{5}\right) \times (-2)$
   d) $\left(-\frac{1}{2}\right) \times \frac{7}{8}$

5. Determine each product. Estimate to place the decimal point.
   a) $(-0.64)(0.2)$
   b) $(-0.5)(-5.71)$
   c) $(-4.13)(-0.8)$
   d) $(0.7)(8.5)$

6. Which of the following expressions have the same product as $\left(-\frac{3}{4}\right)\left(\frac{5}{2}\right)$?
   Explain how you know.
   a) $\left(\frac{5}{2}\right)\left(-\frac{3}{4}\right)$
   b) $\left(\frac{3}{4}\right)\left(-\frac{5}{2}\right)$
   c) $\left(-\frac{3}{2}\right)\left(\frac{5}{4}\right)$
   d) $\left(\frac{3}{4}\right)\left(\frac{5}{2}\right)$
   e) $\left(\frac{3}{2}\right)\left(-\frac{5}{4}\right)$
   f) $\left(-\frac{3}{4}\right)\left(-\frac{5}{2}\right)$

7. Determine each product.
   a) $\left(-\frac{1}{3}\right)\left(\frac{2}{5}\right)$
   b) $\left(\frac{1}{4}\right)\left(-\frac{3}{5}\right)$
   c) $\left(\frac{4}{5}\right)\left(\frac{1}{2}\right)$
   d) $\left(-\frac{5}{6}\right)\left(-\frac{2}{3}\right)$

Apply

8. Suppose each rational number below is multiplied by -2.5.
Which products are greater than 10?
How can you find out by estimating?
Evaluate only those products that are greater than 10.

a) -5.1 **b)** 3.5 **c)** -4.4
d) -3.6 **e)** -5 **f)** 5

9. On February 5, 2008:

a) The price of a share in Petro-Canada changed by $-\$0.80$.
A person owns 120 shares. By how much did the shares change in value that day?

b) The price of a share in Research in Motion changed by $-\$2.10$.
A person owns 50 shares. By how much did the shares change in value that day?

c) The price of a share in Shoppers Drug Mart changed by $\$0.23$.
A person owns 65 shares. By how much did the shares change in value that day?

10. A diver descends at an average speed of 10.4 m/min. Use rational numbers to write her depth after 3.6 min.

11. Determine each product.

a) $(-1.23)(2.8)$ **b)** $(-23.7)(-1.2)$
c) $(15.2)(15.2)$ **d)** $(-20.1)(-5.2)$

12. Determine each product.

a) $\left(\frac{5}{4}\right)\left(-\frac{16}{5}\right)$ **b)** $\left(-\frac{2}{3}\right)\left(-\frac{5}{6}\right)$
c) $\left(-2\frac{8}{9}\right)\left(5\frac{1}{8}\right)$ **d)** $\left(-4\frac{2}{5}\right)\left(-\frac{5}{3}\right)$

13. **Assessment Focus**

a) Multiply: $(-26)(-4)$

b) Use your answer to part a to determine each product.

i) $(-2.6)(-0.4)$
ii) $(-0.26)(0.4)$
iii) $(260)(-0.04)$
iv) $(-0.026)(-4)$

c) Why did you not have to multiply to determine each product in part b?

d) Write 3 more products you could determine using your answer to part a.

14. A courier company has a bank account balance of $45 567.87. The company must repaint all its 25 delivery trucks at an average cost of $3457.25 per truck.

a) Write a multiplication statement with rational numbers to determine the cost of painting the trucks.

b) What is the bank account balance after the bill for painting has been paid? Explain your result.

15. Predict the sign of each product, then calculate the product.

a) $(-2.0)(-0.5)(3.1)$ **b)** $\left(\frac{5}{6}\right)\left(-\frac{4}{7}\right)\left(\frac{3}{2}\right)$

Take It Further

16. Determine the missing number in each product statement.
What strategies did you use?

a) $-3.25 \times \square = 15.275$ **b)** $-\frac{5}{4} \times \square = -\frac{35}{8}$

c) $\square \times 0.045 = -0.018$ **d)** $\square \times 3\frac{3}{4} = 5\frac{1}{4}$

17. A positive rational number is multiplied by a negative rational number. Is it possible that the product is closer to 0 than either of the numbers being multiplied? Explain.

18. Karen used her calculator to evaluate $-\frac{89}{91} \times \frac{31}{86}$. She reported the product as about $-0.352\ 542\ 806$.

a) How did Karen know that the value is approximate?

b) What is the exact answer?

Reflect

Rational numbers can be in fraction form or decimal form. Which form do you prefer to multiply? Explain your choice. Include examples in your explanation.

Math Link

History

When the New York Stock Exchange began in 1792, it modelled its system on the Spanish one. The Spanish dollar was divided into eight parts, so when a stock increased or decreased in value, the change was represented in eighths. A decrease was represented by a negative fraction, while an increase was represented by a positive fraction.
In 2000, the New York Stock Exchange moved to the current system that shows the change in value of a stock as a decimal.
Here are the changes in values of 5 different stocks on a particular day:
$1\frac{5}{8}$, $-\frac{3}{16}$, $\frac{3}{4}$, $-1\frac{7}{16}$, $-1\frac{1}{2}$
Arrange the fractions from least to greatest.
Write each fraction as a decimal to show the change in dollars.

3.5 Dividing Rational Numbers

Marcel has $2\frac{1}{4}$ cups of juice.
He pours $\frac{3}{4}$ of a cup of juice into each glass.
How many glasses can Marcel fill?
Write a division statement to describe this situation.

FOCUS

- Solve problems that require dividing rational numbers.

Investigate

2

➤ The 3rd, 4th, and 5th terms of a number pattern are: +27, −18, +12

- To get the next term, you divide the term before it by one of these rational numbers: $\frac{2}{3}$, $\frac{3}{2}$, $-\frac{2}{3}$, $-\frac{3}{2}$
  Which number is correct? How do you know?
- Determine the 6th and 7th terms of the pattern.
  Describe your strategy and show your work.
- Determine the 1st and 2nd terms of the pattern.
  Describe your strategy and show your work.

➤ Choose a different rational number and a different 1st term.
Calculate the first 5 terms of your pattern.

Reflect & Share

Trade patterns with another pair of classmates.
Write the pattern rule and determine the next 3 terms in your classmates' pattern.
How did you use what you know about rational numbers to identify and extend the pattern?

Connect

To divide 2 rational numbers, use the properties for determining the sign of the quotient of 2 integers to predict the sign of the quotient of the rational numbers. Then:

➤ If the rational numbers are in fraction form:
Use the procedures for dividing 2 fractions to determine the numerical value of the quotient.

➤ If the rational numbers are in decimal form:
Use the procedures for dividing 2 decimals to determine the numerical value of the quotient.

For example,

- When two rational numbers have the same sign, their quotient is positive.
  $\left(-\frac{3}{2}\right) \div \left(-\frac{1}{5}\right) = \frac{15}{2}$ and $\frac{4}{5} \div \frac{3}{2} = \frac{8}{15}$
  $(-3.9) \div (-1.5) = 2.6$ and $9.9 \div 4.5 = 2.2$
- When two rational numbers have opposite signs, their quotient is negative.
  $\frac{3}{2} \div \left(-\frac{1}{5}\right) = -\frac{15}{2}$ and $\left(-\frac{4}{5}\right) \div \frac{3}{2} = -\frac{8}{15}$
  $(-3.9) \div 1.5 = -2.6$ and $9.9 \div (-4.5) = -2.2$

Example 1 Dividing Rational Numbers in Fraction or Mixed Number Form

Determine the sign of each quotient, then divide.

a) $\left(-\frac{5}{8}\right) \div \frac{3}{4}$ b) $\left(-4\frac{1}{5}\right) \div \left(-3\frac{1}{3}\right)$

A Solution

a) $\left(-\frac{5}{8}\right) \div \frac{3}{4}$

The fractions have opposite signs, so their quotient is negative.

Use the strategy of dividing fractions with a common denominator.

Write each fraction with a common denominator of 8.

$\left(-\frac{5}{8}\right) \div \frac{3}{4} = \left(-\frac{5}{8}\right) \div \frac{6}{8}$ Since the denominators are the same, divide the numerators.

$= -\frac{5}{6}$

So, $\left(-\frac{5}{8}\right) \div \frac{3}{4} = -\frac{5}{6}$

b) $\left(-4\frac{1}{5}\right) \div \left(-3\frac{1}{3}\right)$

The mixed numbers have the same sign, so their quotient is positive.

Write each mixed number as an improper fraction: $\left(-\frac{21}{5}\right) \div \left(-\frac{10}{3}\right)$

Use the strategy of multiplying the dividend by the reciprocal of the divisor.

$$\left(-4\frac{1}{5}\right) \div \left(-3\frac{1}{3}\right) = \left(-\frac{21}{5}\right) \div \left(-\frac{10}{3}\right)$$
$$= \left(-\frac{21}{5}\right) \div \left(-\frac{10}{3}\right)$$
$$= \frac{63}{50}$$
$$= 1\frac{13}{50}$$

So, $\left(-4\frac{1}{5}\right) \div \left(-3\frac{1}{3}\right) = 1\frac{13}{50}$

To get the reciprocal of a fraction, interchange the numerator and denominator. So, the reciprocal of $-\frac{10}{3}$ is $-\frac{3}{10}$.

You could use a calculator to divide fractions and mixed numbers. You do not need to input the negative signs if you determine the sign of the quotient first.

When you divide decimals, the quotient may be a terminating or repeating decimal. If you divide using pencil and paper, and the quotient appears to be a repeating decimal, continue to divide until you can identify which digits repeat.

Example 2 Dividing Rational Numbers in Decimal Form

Divide.

a) $(-1.38) \div 0.6$ **b)** $(-0.25) \div (-0.3)$

A Solution

a) $(-1.38) \div 0.6$

Since the dividend and divisor have opposite signs, their quotient is negative.

Estimate first; use compatible numbers.

-1.38 is close to -1, and 0.6 is close to 0.5.

$(-1) \div 0.5$ is -2.

So, $(-1.38) \div 0.6$ is about -2.

Divide integers:
$(-138) \div 6 = -23$
The estimate is -2, so place the decimal point in the quotient between the 2 and the 3.
So, $(-1.38) \div 0.6 = -2.3$

b) $(-0.25) \div (-0.3)$
Since the dividend and divisor have the same sign, their quotient is positive.
Determine the numerical value of the quotient:
$0.25 \div 0.3 = 0.833\,333\ldots$
$= 0.8\overline{3}$
So, $(-0.25) \div (-0.3) = 0.8\overline{3}$

When the divisor is a decimal with more than 1 digit, we use a calculator to divide.

Example 3 Solving Problems Involving Rational Numbers

Determine the missing number in each division statement.

a) $\square \div (-2.6) = 9.62$ **b)** $\left(-\frac{5}{8}\right) \div \square = -\frac{15}{56}$

A Solution

a) $\square \div (-2.6) = 9.62$
Division is the inverse of multiplication.
Any division statement can be written as an equivalent multiplication statement.
Estimate.
Think: $\square \div (-3) = 9$
We know that $(-27) \div (-3) = 9$, or as a multiplication statement: $(-27) = (-3) \times 9$
Rewrite the given statement the same way:
$\square \div (-2.6) = 9.62$ can be written as $\square = (-2.6) \times 9.62$
Use a calculator: $\square = -25.012$
The missing number is -25.012.

b) $\left(-\frac{5}{8}\right) \div \square = -\frac{15}{56}$

$\left(-\frac{5}{8}\right) \div \square = -\frac{15}{56}$ can be written as $\square = \left(-\frac{5}{8}\right) \div \left(-\frac{15}{56}\right)$

The quotient is positive.

Use the strategy of multiplying by the reciprocal to determine the numerical value of the quotient.

$\square = \frac{5}{8} \div \frac{15}{56}$

$= \frac{5}{8} \times \frac{56}{15}$ Simplify by dividing by common factors in the numerator and denominator.

$= \frac{{}^{1}\cancel{5}}{{}_{1}\cancel{8}} \times \frac{\cancel{56}^{7}}{\cancel{15}_{3}}$

$= \frac{1}{1} \times \frac{7}{3}$

$= \frac{7}{3}$

The missing number is $\frac{7}{3}$.

Solve a simpler problem.
Think: $6 \div \square = 2$
We know $6 \div 3 = 2$,
so we write the related statement:
$3 = 6 \div 2$

Discuss the ideas

1. How can you use what you know about dividing integers and dividing fractions to divide 2 rational numbers in fraction form?

2. How can you use what you know about dividing integers and dividing decimals to divide 2 rational numbers in decimal form?

Practice

Check

3. Predict the sign of each quotient, then calculate the quotient.

a) $(-1.5) \div 3$ **b)** $2.8 \div (-2)$

c) $(-8.4) \div (-4)$ **d)** $1.6 \div (-8)$

e) $(-14.4) \div (-6)$ **f)** $(-6.3) \div 7$

4. Predict the sign of each quotient, then calculate the quotient.

a) $\frac{1}{2} \div \left(-\frac{3}{4}\right)$ **b)** $\left(-\frac{2}{5}\right) \div \frac{3}{10}$

c) $\left(-\frac{7}{6}\right) \div \left(-\frac{8}{3}\right)$ **d)** $\frac{1}{4} \div \frac{11}{3}$

e) $\frac{5}{2} \div \left(-\frac{2}{3}\right)$ **f)** $\left(-\frac{9}{5}\right) \div \left(-\frac{11}{4}\right)$

5. Which of the following expressions have the same answer as $\left(-\frac{1}{3}\right) \div \left(-\frac{3}{4}\right)$?

a) $\left(-\frac{1}{3}\right) \times \left(-\frac{3}{4}\right)$ **b)** $\left(-\frac{3}{4}\right) \div \left(-\frac{1}{3}\right)$

c) $\left(-\frac{1}{3}\right) \times \left(-\frac{4}{3}\right)$ **d)** $\left(-\frac{4}{3}\right) \times \left(-\frac{1}{3}\right)$

e) $\frac{1}{3} \div \frac{3}{4}$ **f)** $\frac{4}{3} \times \frac{1}{3}$

Apply

6. At a sea port, the effect of the tide changed the water level by -5.6 m in 3.5 h.
What was the mean change in water level per hour?

7. Determine each quotient without a calculator. Estimate to place the decimal point in the quotient.

a) $0.32 \div 0.4$

b) $(-1.17) \div 0.8$

c) $0.25 \div (-0.6)$

d) $(-1.02) \div (-0.2)$

e) $3.76 \div (-0.3)$

f) $3.15 \div 0.9$

8. On a winter's day, the temperature at 6 P.M. was 0°C. Suppose the temperature decreased by 2.5°C each hour until it was -12.5°C. How long did it take to reach this temperature? How do you know?

9. Use a calculator to determine each quotient.

a) $20.736 \div (-1.8)$

b) $(-27.94) \div 1.2$

c) $(-84.41) \div (-2.3)$

d) $23.04 \div 4.8$

e) $76.63 \div (-7.5)$

f) $(-0.1081) \div 0.45$

10. Assessment Focus Suppose each rational number below is divided by -0.5.
Predict which quotients are greater than -10.
Explain the strategies you used to predict.
Then evaluate only those quotients that are greater than -10.

a) -20.5 **b)** 18.8 **c)** 10.7 **d)** 0.6

11. To pay for a skiing holiday in Whistler, Paige borrowed \$1450.50 from her parents. She pays back \$30.75 each week.

a) How many weeks will it be until Paige is no longer in debt? Justify your answer.

b) How did you use rational numbers to calculate the answer in part a?

12. Determine each quotient.

a) $\frac{5}{4} \div \left(-\frac{7}{6}\right)$ **b)** $\frac{3}{10} \div \frac{12}{5}$

c) $\left(-\frac{3}{4}\right) \div \left(-1\frac{1}{8}\right)$ **d)** $\left(-4\frac{3}{5}\right) \div \frac{3}{4}$

e) $3\frac{2}{3} \div \left(-2\frac{1}{4}\right)$ **f)** $3\frac{4}{9} \div 6\frac{1}{3}$

13. A thermometer on a freezer is set at -5.5°C. Each time the freezer door is opened, the temperature increases by 0.3°C.
Suppose there is a power outage.
How many times can the door be opened before the temperature of the freezer increases to 5°C? Justify your solution.

14. On one day in January, the temperature changed by $-15.4°C$ in 5.5 h.
What was the mean change in temperature per hour?

15. A person has 54 shares in WestJet Airlines. On February 6, 2008, these shares lost $17.28 in value.
What was the change in value of 1 share? How do you know?

16. Suppose each rational number below was divided by $-\frac{2}{3}$. Predict which quotients would be less than $-\frac{1}{2}$. Explain the strategy you used to predict.

a) $-\frac{2}{3}$ **b)** $\frac{1}{3}$

c) $\frac{5}{6}$ **d)** $\frac{1}{4}$

17. Determine the missing number in each division statement.

a) $\square \div 1.25 = -3.6$

b) $\square \div \left(-\frac{3}{4}\right) = \frac{7}{8}$

c) $(-0.5875) \div \square = -0.25$

d) $\frac{68}{15} \div \square = -\frac{4}{5}$

18. Replace each $\square$ with a rational number to make each equation true. Explain the strategy you used.

a) $(-0.3) \times \square = 0.78$

b) $0.8 \times \square = -5.52$

c) $(-1.26) \div \square = 0.2$

d) $\square \div (-1.1) = 3.26$

Take It Further

19. Alex and Ellice run in opposite directions from school to their homes.
Ellice runs 1.3 km to her home in 7.8 min.
Alex runs 630 m to his home in 4.2 min.

a) Write division statements using positive and negative rational numbers to represent each student's average speed in metres per minute.
What do the positive and negative numbers represent?

b) Who runs at the greater average speed?

20. Write 6 division statements that have a quotient between $-\frac{3}{4}$ and $-\frac{1}{4}$.

21. Which expression below has the greatest value? How can you find out without calculating every answer?

a) $-\frac{1}{2} + \left(-\frac{2}{3}\right)$ **b)** $-\frac{1}{2} - \left(-\frac{2}{3}\right)$

c) $\left(-\frac{1}{2}\right) \times \left(-\frac{2}{3}\right)$ **d)** $\left(-\frac{1}{2}\right) \div \left(-\frac{2}{3}\right)$

Reflect

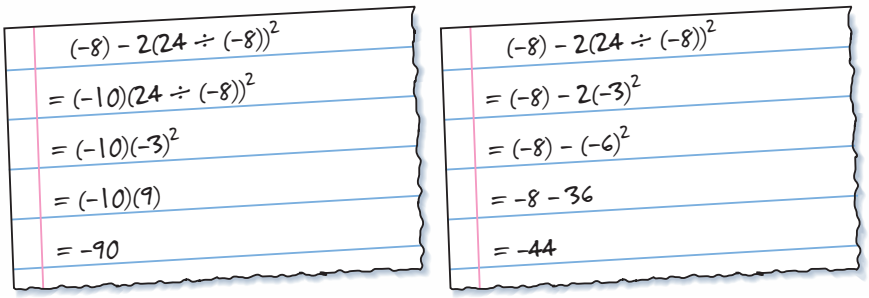
How is dividing rational numbers similar to multiplying them?
Include examples of fractions and decimals in your explanation.

3.6 Order of Operations with Rational Numbers

FOCUS

- Explain and apply the order of operations with rational numbers.

Two students were asked to evaluate: $(-8) - 2(24 \div (-8))^2$
Here are their calculations.



Why did both these students get incorrect answers?
What is the correct answer?

Investigate

2

Use a calculator when you need to.
Use any operations or brackets with these rational numbers: -2.1, -0.5, 3.4, 0.9

➤ Write an expression and determine its value.
➤ Try to find an expression with a greater value. What strategies did you use to do this?
➤ Repeat this activity several times. Which expression has the greatest value?

Reflect & Share

Compare your expression with the greatest value with that of another pair of students. Are the expressions the same? If not, whose expression has the greater value? Share your strategies.
Work together to find the expression with the least value.

Connect

In Lesson 3.1, you learned that integers and fractions are rational numbers. So, the order of operations for all rational numbers is the same as that for integers and fractions:

- Do the operations in brackets first.
- Do any work with exponents.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

Example 1 Using the Order of Operations with Decimals

Evaluate.

a) $(-0.8) + 1.2 \div (-0.3) \times 1.5$ **b)** $(-3.2) - 0.9 \div [0.7 - (-1.2)]^2$

A Solution

a)
$(-0.8) + 1.2 \div (-0.3) \times 1.5$ Divide first: $1.2 \div (-0.3) = -4$
$= (-0.8) + (-4) \times 1.5$ Then multiply: $(-4) \times 1.5 = -6$
$= (-0.8) + (-6)$ Then add.
$= -6.8$

b)
$(-3.2) - 0.9 \div [0.7 - (-1.2)]^2$ Subtract in the brackets first: add the opposite.
$= (-3.2) - 0.9 \div [0.7 + 1.2]^2$ Add: $0.7 + 1.2 = 1.9$
$= (-3.2) - 0.9 \div [1.9]^2$ Use a calculator to evaluate the power: $[1.9]^2 = 3.61$
$= (-3.2) - 0.9 \div 3.61$ Then divide: $0.9 \div 3.61 \doteq 0.249\,307\,479$
$\doteq -3.2 - 0.249\,307\,479$
$\doteq -3.449\,307\,479$

Since the answer does not terminate or appear to repeat, round the answer to the nearest tenth because the numbers in the question are given in that form.

So, $(-3.2) - 0.9 \div [0.7 - (-1.2)]^2 \doteq -3.4$

After we substitute rational numbers for variables in a formula, we simplify the numerical expression using the order of operations.

When you evaluate with decimals, use a calculator when the divisor has more than 1 digit and when the number you multiply by has more than 2 digits.

Example 2 Solving Problems Using the Order of Operations

To convert a temperature in degrees Fahrenheit to degrees Celsius, we use the formula:

$C = \frac{F - 32}{1.8}$

In Fort Simpson, Northwest Territories, the mean temperature in December is -9.4°F. What is this temperature in degrees Celsius?

A Solution

Substitute $F = -9.4$ into the formula:

$$C = \frac{F - 32}{1.8}$$
$$= \frac{-9.4 - 32}{1.8}$$

The fraction bar indicates division, but also acts like brackets.

That is, the expression means $C = (-9.4 - 32) \div 1.8$

So, simplify the numerator first, then divide.

$$C = \frac{-9.4 - 32}{1.8} \qquad \text{Subtract.}$$
$$= \frac{-41.4}{1.8} \qquad \text{Divide.}$$
$$= -23$$

The mean temperature in December is -23°C.

Example 3 Using the Order of Operations with Fractions

Evaluate.

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{1}{3} + \left(-\frac{3}{12}\right)\right]$$

A Solution

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{1}{3} + \left(-\frac{3}{12}\right)\right] \qquad \text{Add in the square brackets first.}$$
$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{1}{3} - \frac{3}{12}\right] \qquad \text{Use a common denominator of 12.}$$
$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{4}{12} - \frac{3}{12}\right]$$
$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left(\frac{1}{12}\right) \qquad \text{Multiply next.}$$
$$= \frac{1}{4} - \left(-\frac{2}{3}\right) \div \left(\frac{1}{12}\right) \qquad \text{Then divide: multiply by the reciprocal of the divisor.}$$
$$= \frac{1}{4} - \left(-\frac{2}{\cancel{3}_1}\right) \times \left(\frac{\cancel{12}^4}{1}\right)$$
$$= \frac{1}{4} - \left(-\frac{2}{1}\right) \times \left(\frac{4}{1}\right)$$
$$= \frac{1}{4} - (-8)$$
$$= \frac{1}{4} + 8$$
$$= 8\frac{1}{4}$$

Discuss the ideas

1. What does a fraction bar indicate?

2. As the number of operations increases and the expressions become more complex, it is easy to make mistakes.
What can you do to prevent yourself making mistakes?

Practice

Check

3. Evaluate. Do not use a calculator.

a) $2.3 - (-1.6) \times (0.8)$

b) $(-14.8) \times 0.9 - 3.1$

c) $(-12.8) \div (-0.2) + 4.5 \div 0.5$

d) $(-4.8) \times (-0.4 + 0.6)^2$

4. Evaluate. Do not use a calculator.

a) $\frac{1}{2} + \left(-\frac{3}{4}\right) \times \frac{1}{3}$

b) $\left(-\frac{5}{4}\right) \div \left(-\frac{1}{4} + \frac{3}{2}\right)\left(-\frac{1}{4} + \frac{3}{2}\right)$

c) $\left(-\frac{7}{10}\right) \div \left(-\frac{2}{5}\right) - \left(-\frac{1}{4}\right) \times \frac{1}{2}$

d) $\frac{6}{5} \times \left(-\frac{2}{3} + \frac{8}{3}\right)^2 - \frac{5}{12}$

Apply

5. a) Use a calculator to evaluate the expression below. Key in the expression as it is written.
$-2.8 - 1.4 \times 4.5$

b) Does the calculator follow the order of operations or does it perform operations from left to right? How did you find out?

6. Estimate which expression has the greatest value. Then use a calculator to evaluate each expression to verify your prediction.

a) $9.1 - 3.5 \times (4.2)^2$ b) $(9.1 - 3.5) \times (4.2)^2$

c) $9.1 - (3.5 \times 4.2)^2$ d) $9.1[(-3.5) \times (4.2)^2]$

7. Evaluate.

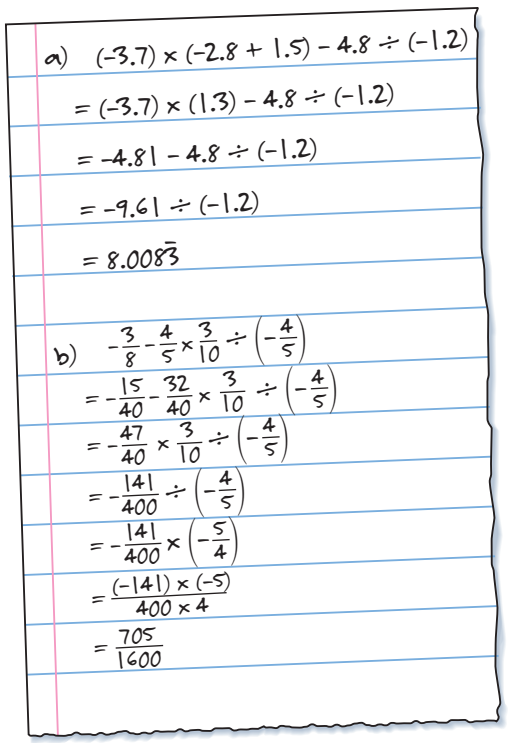
a) $\left(-\frac{2}{3}\right) \div \frac{1}{4} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3}$

b) $\left(-\frac{2}{3}\right) \div \left[\frac{1}{4} + \left(-\frac{1}{2}\right)\right] \times \frac{1}{3}$

c) $\left(-\frac{2}{3}\right) \div \left[\frac{1}{4} - \left(-\frac{1}{2}\right)\right] \times \frac{1}{3}$

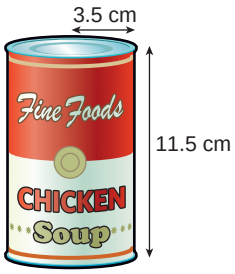
d) $\left(-\frac{2}{3}\right) \div \left[\frac{1}{4} + \left(-\frac{1}{2}\right) \times \frac{1}{3}\right]$

8. Find the errors in each solution.
Write the correct solution.



9. A family moves from Chicago to Saskatoon. A company that rents moving trucks uses this formula, $C = 1.15[21.95d + 0.035(k - 120)]$, to determine the cost, including tax, of renting a truck for d days and k kilometres, when $k > 120$. The distance from Chicago to Saskatoon is 2400 km and the family travels for 4 days.
What is the cost to rent the truck?

10. A can of soup is a cylinder with radius 3.5 cm and height 11.5 cm.



Use the formula:
Surface area $= 2\pi r^2 + 2\pi r \times$ height,
where r is the radius of the can

a) Determine the area of tin needed to make the can, to the nearest square centimetre.

b) Explain how you used the order of operations in part a.

11. **a)** Use this formula to convert each Fahrenheit temperature below to Celsius:
$C = \frac{F - 32}{1.8}$

i) 0°F **ii)** −40°F **iii)** −53°F

b) Here is another way to write the formula in part a: $C = \frac{5}{9}(F - 32)$
Use this formula to convert each Fahrenheit temperature below to Celsius:

i) 50°F **ii)** −13°F **iii)** 32°F

c) Which formula in parts a and b was easier to use? Explain your choice.

12. Evaluate. State the order in which you carried out the operations.

a) $\left(-4\frac{1}{2}\right) + \left(-\frac{2}{3}\right) \times 2\frac{3}{4}$

b) $\left(-3\frac{2}{5}\right) \times \left(-1\frac{5}{6}\right) + \frac{3}{10}$

c) $(-3) \div \left(-\frac{4}{5}\right) + \left(-\frac{5}{12}\right) \times 1\frac{1}{2}$

d) $\left(1\frac{5}{8}\right) - \left(-2\frac{3}{4} + 2\right)\left(-2\frac{3}{4} + 2\right)$

13. Use a calculator to evaluate.
Write the answers to the nearest hundredth where necessary.

a) $2.3 + (-11.2) \div (-0.2) - 3.7$

b) $(-3.4) \times 0.7 - (-1.8)(-1.8)$

c) $\frac{0.67 - 4.2 \div (-0.2)}{(-7.3 + 8.6)^2}$

d) $\frac{8.9 \times (-3.1 + 22.7)^2 + 4.7}{(-9.6) \div 0.04 - 0.4}$

14. On one day in Black Lake, Saskatchewan, the maximum temperature was −8.1°C and the minimum temperature was −16.7°C.

a) What was the mean temperature that day?

b) How did you use the order of operations in part a?

15. **Assessment Focus** Use these numbers to make 4 fractions: 2, −3, 4, −5, 6, −8, 10, −12

a) Use the 4 fractions to write an expression using 3 different operations and brackets. Evaluate the expression.

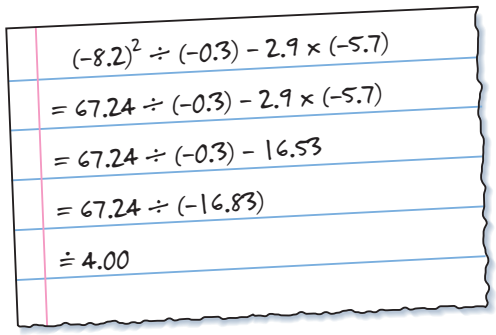
b) Use the same 4 fractions a different way or use 4 different fractions. Write an expression whose value is as close to 0 as possible. Show your work.

16. The following maximum temperatures were recorded for one week in Abbotsford, BC: −3.1°C, −4.5°C, −6.2°C, −1.2°C, 1.5°C, 2.3°C, 4.1°C

a) Predict whether the mean maximum temperature for the week is above or below 0°C.

b) Calculate the mean maximum temperature for the week.

17. A student's solution to a problem, to the nearest hundredth, is shown below. The solution is incorrect. Identify the errors. Provide a correct solution.



18. A student evaluated the following expression and the answer was 50.39 to the nearest hundredth. Another student evaluated the expression and the answer was 1.63 to the nearest hundredth.

$$\frac{23.7 - (-5.6) \div 0.7 + 6.8}{(-3) \times (-6.7) + 3.5}$$

a) Which answer is correct?

b) What mistake did one student likely make?

Take It Further

19. In question 11, you used these two versions of a formula to convert Fahrenheit temperatures to Celsius:

$C = \frac{F - 32}{1.8}$ and $C = \frac{5}{9}(F - 32)$

Explain how to get one version of the formula from the other.

20. In Flin Flon, Manitoba, the mean of the maximum and minimum temperatures on one day was −12.8°C. The maximum temperature was −11.5°C.
What was the minimum temperature?

21. Insert brackets in the expression below so the statement is correct.
Is it possible to insert brackets and get a positive answer?
Explain your thinking.

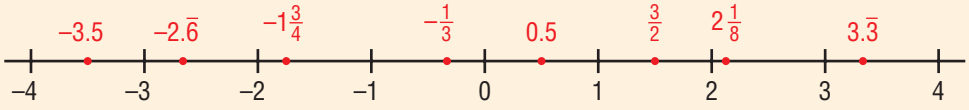
$-3.8 + 9.1 \times -2.5 \; -0.5 = -31.1$

Reflect

When you use the order of operations with rational numbers, do you prefer to work with the numbers in decimal form or fraction form? Explain your choice.

Study Guide

A rational number is any number that can be written in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$.
This number line illustrates some different forms of rational numbers:

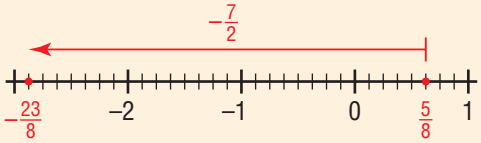


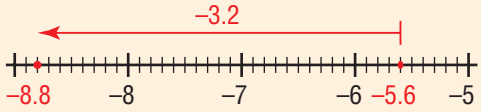
From least to greatest: $-3.5, -2.\overline{6}, -1\frac{3}{4}, -\frac{1}{3}, 0.5, \frac{3}{2}, 2\frac{1}{8}, 3.\overline{3}$

To operate with rational numbers, apply what you know about operating with fractions, decimals, and integers.

- To add rational numbers, visualize a number line.

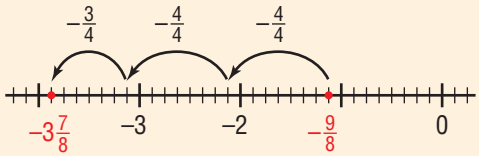
$\frac{5}{8} + \left(-\frac{7}{2}\right) = -\frac{23}{8}$ $\qquad$ $(-5.6) + (-3.2) = -8.8$

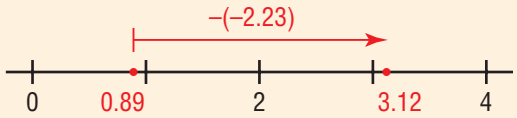




- To subtract rational numbers, visualize a number line.

$-\frac{9}{8} - \frac{11}{4} = -\frac{31}{8}$ $\qquad$ $0.89 - (-2.23) = 3.12$





- To multiply rational numbers, determine the sign of the product first.

$\left(\frac{3}{4}\right)\left(-\frac{5}{2}\right) = -\frac{15}{8}$ $\qquad$ and $\qquad$ $(-4.13)(-0.8) = 3.304$

- To divide rational numbers, determine the sign of the quotient first.

$\left(-\frac{3}{10}\right) \div \left(-\frac{12}{5}\right) = \frac{1}{8}$ $\qquad$ and $\qquad$ $76.63 \div (-7.5) = -10.217\overline{3}$

The order of operations with rational numbers is the same as the order for whole numbers, fractions, and integers:

- Do the operations in brackets first.
- Then evaluate the exponents.
- Then divide and multiply, in order, from left to right.
- Then add and subtract, in order, from left to right.

Review

3.1

1. Which of the following rational numbers are between -2.5 and $-\frac{11}{3}$? How do you know?

a) -3.4 **b)** $-\frac{9}{4}$ **c)** $-\frac{19}{6}$ **d)** -4.2

2. Order the following rational numbers from least to greatest. Show them on a number line.

$3.12, -\frac{4}{3}, 0.9, -\frac{1}{2}, -0.4$

3. Write 3 rational numbers between each pair of numbers. Sketch number lines to show all the rational numbers.

a) $-3.5, -3.1$ **b)** $\frac{1}{5}, \frac{7}{10}$

c) $0.8, 0.9$ **d)** $-\frac{5}{2}, -\frac{3}{2}$

4. On one day, the prices of 5 stocks changed by the following amounts in dollars:

$-0.09, -0.51, +0.95, +0.54, -2.00$

Order the amounts from the greatest loss to the greatest gain.

3.2

5. Determine each sum.

a) $-1.2 + (-0.3)$

b) $134.89 + (-56.45)$

c) $-23.6 - 4.57$

d) $48.05 + 0.003$

6. A technician checked the temperature of a freezer and found that it was $-15.7°C$. She noted that the temperature had dropped $7.8°C$ from the day before.

a) What was the temperature the day before?

b) Show both temperatures on a vertical number line.

7. Determine each sum.

a) $\frac{3}{4} + \frac{7}{8}$ **b)** $-1\frac{1}{2} + 3\frac{1}{3}$

c) $-4\frac{5}{6} + \left(-1\frac{5}{12}\right)$ **d)** $\frac{11}{9} + \left(-\frac{17}{6}\right)$

3.3

8. Determine each difference.

a) $-3.4 - (-4.8)$

b) $-71.91 - 11.23$

c) $90.74 - 100.38$

d) $63.2 - 80.02$

9. At the end of a day, the price of a stock was \$21.60. During the day, the price of the stock had changed by $-\$0.75$. What was the price of the stock at the beginning of the day? How do you know?

10. Determine each difference.

a) $\frac{4}{3} - \frac{11}{6}$ **b)** $-\frac{5}{8} - \left(-\frac{7}{5}\right)$

c) $3\frac{5}{7} - \left(-6\frac{9}{10}\right)$ **d)** $-\frac{23}{4} - \frac{23}{3}$

3.4

11. Predict which expressions have a value between -1 and 1. Calculate each product to check.

a) $(-1.4) \times (-0.8)$ **b)** $25.6 \times (-0.05)$

c) $\left(-\frac{3}{5}\right)\left(\frac{4}{3}\right)$ **d)** $\left(-\frac{5}{6}\right)\left(-\frac{2}{3}\right)$

12. The temperature in Richmond, BC, at 4:00 P.M. was 2°C. The temperature drops 1.3°C each hour. What will the temperature be at 11:00 P.M.? Justify your answer.

13. Write 3 multiplication statements that have the same product as $\left(-\frac{4}{9}\right)\left(\frac{7}{5}\right)$. How can you check your answers?

14. Determine each product.

a) $3.5 \times (-0.3)$ **b)** $(-4.1)(2.3)$

c) $\left(-\frac{4}{7}\right)\left(-\frac{2}{3}\right)$ **d)** $1\frac{3}{5} \times \left(-2\frac{1}{2}\right)$

15. A mountain climber descends from base camp at an average speed of 5.9 m/h. How far below base camp will the climber be after 3.75 h? Use a vertical number line with the base camp at 0 to illustrate the climber's descent.

3.5

16. Predict which expressions have a value between -1 and 1. Calculate each quotient to check.

a) $(-2.2) \div 0.4$ **b)** $10.6 \div (-9.2)$

c) $\frac{9}{10} \div \left(-\frac{3}{2}\right)$ **d)** $\left(-\frac{5}{12}\right) \div \left(-\frac{5}{4}\right)$

17. Write 3 division statements that have the same quotient as $\frac{3}{8} \div \left(-\frac{5}{11}\right)$.

18. Replace each $\square$ with a rational number to make each equation true. Explain the strategy you used.

a) $(-0.2) \times \square = 0.75$

b) $0.9 \times \square = -7.47$

c) $(-0.624) \div \square = -0.4$

19. Determine each quotient.

a) $8.4 \div (-1.2)$ **b)** $(-20.6) \div (-0.9)$

c) $\left(-\frac{9}{11}\right) \div \left(\frac{7}{5}\right)$ **d)** $\left(-1\frac{2}{3}\right) \div 3\frac{1}{2}$

3.6

20. **a)** Evaluate each expression. Do not use a calculator.

i) $-3.5 + 6.2 \times (-0.2)$

ii) $(-3.5 + 6.2) \times (-0.2)$

b) Are the answers in part a different? Explain.

21. Predict whether the value of each expression below is positive or negative. Explain how you predicted. Evaluate to check your prediction.

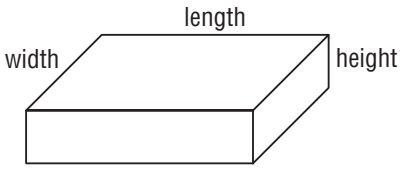
a) $-\frac{3}{5} + \left[\frac{1}{3} \times \left(-\frac{3}{4}\right)\right]$

b) $\left(-\frac{3}{5} + \frac{1}{3}\right) \times \left(-\frac{3}{4}\right)$

c) $-\left(-\frac{3}{5} + \frac{1}{3}\right) \times \left(-\frac{3}{4}\right)$

22. A formula for the surface area of a right rectangular prism is:

2(length $\times$ width + length $\times$ height + width $\times$ height)



a) Determine the surface area of a right rectangular prism with length 25.3 cm, width 15.2 cm, and height 9.7 cm.

b) Explain how you used the order of operations in part a.

23. Evaluate each expression. Show your work to illustrate the order of operations.

a) $-1.2 \div (0.6) - [6.3 + (-3.4)]$

b) $-\frac{5}{12} + \left(\frac{4}{3}\right)\left(\frac{4}{3}\right)$

c) $-\frac{4}{5} \div \left[\frac{1}{2} + \left(-\frac{1}{6}\right)\left(-\frac{1}{6}\right) \times \frac{1}{4}\right]$

d) $\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) \div \frac{2}{9} - \left(-\frac{4}{5}\right)$

e) $-1\frac{3}{7} \times \frac{1}{2} + \left(-3\frac{1}{7}\right)$

f) $0.2 - (-1.2) \times 0.5 \div (-0.1)$

g) $(-0.2 + 0.9)^2 + 9.8 \div (-0.7)$

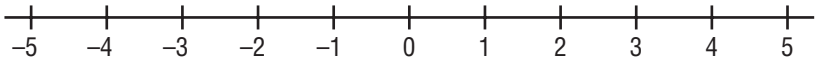
Practice Test

1. **a)** Identify a rational number between -0.5 and -0.6.

b) How do you know the number you identified in part a is a rational number?

2. **a)** Write the following rational numbers on a copy of the number line below:

$0.6,\ -0.\overline{3},\ -2.5,\ 3.\overline{6},\ 4\frac{1}{2},\ -1\frac{3}{10},\ -\frac{23}{5},\ \frac{11}{3}$



b) List the numbers in part a from greatest to least.

3. Evaluate.

a) $-7.4 - (-6.1)$ **b)** $\frac{4}{5} + \left(-\frac{3}{10}\right)$ **c)** $(-3.2)(-0.5)$ **d)** $\left(-\frac{3}{4}\right) \div \frac{1}{3}$

4. Sarah has a balance of $-\$2.34$ in her account.

Each time she makes a withdrawal, she is charged \$1.20.

a) What does "a balance of $-\$2.34$" mean?

b) Sarah makes three more withdrawals of \$20.50 each.

What is her balance now?

How can you use rational numbers to calculate it?

c) Sarah's overdraft limit is \$500.00. How many more \$20.50 withdrawals can she make? Justify your answer.

5. Evaluate. How could you check your answers?

a) $(-56.8)(-14.5)$ **b)** $\left(-3\frac{1}{3}\right)\left(-2\frac{3}{10}\right)$

c) $\left(-4\frac{2}{5}\right) \div \left(-1\frac{5}{7}\right)$ **d)** $45.8 \div (-12.2)$

6. **a)** A student evaluated the expression below and got the answer 1.

What is the correct answer? How do you know?

$\frac{1}{2} + \left(-\frac{3}{4}\right) \div \left(-\frac{1}{4}\right)$

b) What might the student have done wrong to get the answer 1?

7. Evaluate. Use a calculator when you need to.

a) $-3.1 + 4.5 \times (-2.9) - 7.2 \div (-3)$

b) $(-9.7) \times (-1.2) + 5.4^2 \div (-3.6)$

Unit Problem Investigating Temperature Data

The table shows the monthly lowest temperature, in degrees Celsius, in Edmonton.

Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
−14.7	−12.8	−7.2	0.5	5.9	10.1	12.3	10.6	6.1	−0.8	−8.3	−13.4

1. Determine the mean monthly lowest temperature in Edmonton for that year.

a) How did you use the order of operations in your calculation?

b) Which months have temperatures below the mean? How do you know?

Here are the monthly highest temperatures in Edmonton for the same year.

Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
−5.8	−2.9	2.1	10.6	17.2	20.7	23.4	22.2	16.7	9.8	1.1	−4.3

2. Determine the mean monthly highest temperature in Edmonton for that year.

3. Choose three months. For each month, calculate the difference between:

a) the mean monthly highest temperature and the monthly highest temperature

b) the mean monthly lowest temperature and the monthly lowest temperature

4. Some climatologists predict that, by the end of the century, due to global warming, the mean temperature will increase by between 1.4°C and 11°C.

a) How might the mean monthly highest temperature be affected? In what range could this temperature lie in 2100?

b) Repeat part a for the mean monthly lowest temperature.

Your work should show:

- your calculations for the mean lowest and highest temperatures
- your calculations of the differences in temperatures
- the possible ranges for temperatures in the year 2100

Reflect on Your Learning

What is the most important thing you have learned about rational numbers? Explain why it is important.

Cumulative Review — Units 1–3

1

1. Determine the value of each square root.

a) $\sqrt{\frac{1}{25}}$ b) $\sqrt{\frac{225}{169}}$ c) $\sqrt{\frac{9}{121}}$

d) $\sqrt{1.44}$ e) $\sqrt{0.16}$ f) $\sqrt{3.24}$

2. Determine the side length of a square with each area below. Explain your strategy.

a) 64 cm^2

b) 1.21 m^2

c) 72.25 mm^2

3. Calculate the number whose square root is:

a) 0.7 b) 1.6 c) 0.006

d) $\frac{12}{17}$ e) $\frac{1}{3}$ f) $\frac{2}{13}$

4. Which decimals and fractions are perfect squares? Explain your reasoning.

a) $\frac{7}{63}$ b) $\frac{12}{27}$ c) $\frac{4}{18}$

d) 0.016 e) 4.9 f) 0.121

5. A square garden has area 6.76 m^2.

a) What is the side length of the garden?

b) One side of the garden is against a house. How much fencing is needed to enclose the garden? How do you know?

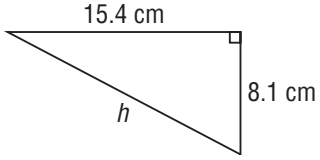
6. Determine 2 decimals that have square roots from 12 to 13.

7. Use any strategy you wish to estimate the value of each square root.

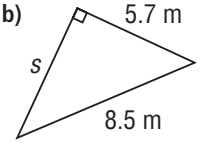
a) $\sqrt{\frac{1}{35}}$ b) $\sqrt{\frac{65}{4}}$ c) $\sqrt{0.8}$ d) $\sqrt{0.11}$

8. Determine the unknown length in each triangle to the nearest tenth.

a)



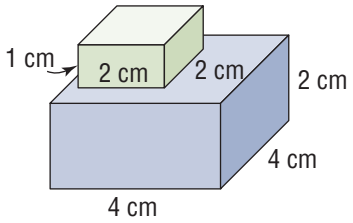
b)



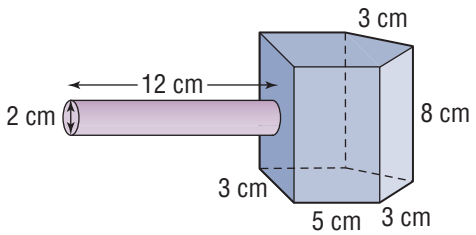
9. This object is built with 1-cm cubes. Determine its surface area.

10. Determine the surface area of each composite object. Give the answers to the nearest whole number. Explain how you accounted for the overlap in each case.

a)



b)



2

11. Write each product as a power, then evaluate.

a) $4 \times 4 \times 4$

b) $6 \times 6 \times 6 \times 6$

c) $(-3)(-3)(-3)(-3)(-3)(-3)(-3)$

d) $-(-2)(-2)(-2)(-2)(-2)(-2)(-2)$

e) $-(10 \times 10 \times 10 \times 10 \times 10)$

f) $-(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)$

12. Predict the sign of each answer, then evaluate.

a) $-(-3)^4$ b) $(-5)^6$ c) -4^3

d) $(-7)^2$ e) -7^0 f) $(-10)^0$

13. Write each number using powers of 10.

a) 800 b) 52 000

c) 1760 d) 7 000 004

14. Evaluate.

a) $[3 \times (-2)^3 - 4]^2$

b) $(-7 + 5)^2 - [4 + (-1)^3]^2$

c) $9^2 \div (-3)^3 + 5^2 - 2^5$

d) $(3^7 - 2^{11})^1 \div (4^7 + 3^8)^0$

e) $(-4)^2 - 3^3 + (-2)^4 - 1^5$

f) $[8^4 \div (-4)^6 \times 2^0]^{10}$

15. Express as a single power.

a) $6^5 \times 6^{11} \div 6^8$

b) $(-3)^6 \div (-3)^2 \times (-3)^4$

c) $\dfrac{(-5)^6 \times (-5)^9}{(-5)^7 \times (-5)^5}$

d) $\dfrac{2^8}{2^2} \times \dfrac{2^{12}}{2^4}$

16. Evaluate.

a) $7^2 - 4^3 \times 4^0 + 3^2$

b) $(-2)^8 \div (-2)^4 - (-2)^7 \div (-2)^5$

c) $-5^2(5^4 \div 5) - 5^3$

d) $\dfrac{8^{13} \times 8^{14}}{8^{15} \times 8^9}$

17. A wheat field is 10 000 m wide. The area of the field is 10^8 m^2.

a) Use the exponent laws to determine the length of the field.

b) What is the perimeter of the field? Did you use any exponent laws to calculate the perimeter? Explain.

18. Simplify, then evaluate each expression.

a) $(6^2)^8 \div (6^4)^2$

b) $(7^4 \div 7^2)^3 + (3^5 \div 3^2)^3$

c) $[(-2)^5 \div (-2)^4]^3 - [(-5)^2 \times (-5)^3]^0$

d) $(4 \times 9)^4 + (3^5)^2$

e) $[(-4)^3]^2 - [(-2)^4]^3 - [(-3)^2]^4$

f) $[9 \div (-3)]^2 \times 3^4$

3

19. Show each set of numbers on a number line. Order the numbers from least to greatest.

a) $-1.9, -3.3, 4.8, -2.8, 1.2, -3.\overline{3}$

b) $\frac{19}{5}, -\frac{13}{4}, \frac{3}{4}, -2\frac{1}{2}, -\frac{13}{10}, -\frac{2}{5}$

c) $1.1, \frac{4}{3}, -\frac{1}{3}, -1.01, 1\frac{3}{8}, -0.11$

d) $\frac{2}{9}, -0.2, 0.25, -\frac{1}{6}, -0.\overline{1}, \frac{1}{8}$

20. Determine each sum or difference.

a) $17.4 + (-15.96)$ b) $-8.38 + (-1.927)$

c) $-4.5 - (-13.67)$ d) $13.28 - 19.71$

e) $-\frac{3}{4} + \frac{2}{3}$ f) $1\frac{5}{8} + \left(-6\frac{1}{3}\right)$

g) $-\frac{17}{4} - \frac{11}{3}$ h) $3\frac{5}{6} - \left(-2\frac{2}{3}\right)$

21. The changes in value of a stock were recorded in the table below.

Day	Change in Value ($)
Monday	–0.450
Tuesday	0.327
Wednesday	–0.065

The price of the stock by the end of the day on Wednesday was $85.460. Use rational numbers to calculate the price of the stock on Monday morning.

22. Determine each product or quotient.

a) $(-14.6)(2.5)$ b) $(-12.8)(-12.8)$

c) $(-8.64) \div (-2.7)$ d) $4.592 \div (-0.82)$

e) $\left(\frac{9}{5}\right)\left(6\frac{1}{3}\right)$ f) $\left(-8\frac{3}{4}\right)\left(2\frac{2}{15}\right)$

g) $\left(-\frac{5}{12}\right) \div \left(-8\frac{1}{3}\right)$ h) $\left(-3\frac{1}{5}\right) \div 2\frac{2}{3}$

23. Evaluate.

a) $\left(-\frac{7}{8}\right) - \frac{1}{5} \div \left(-\frac{3}{10}\right) - \frac{1}{4}$

b) $(-2.1)(18.5) - 6.8 \div 4$

c) $\left(-7\frac{1}{3}\right)\left(\frac{6}{55}\right) + 1\frac{1}{2} \div \left(-\frac{2}{7}\right)$

d) $2\frac{1}{4} - \left(-3\frac{7}{8} + 5\right)\left(\frac{4}{9} - 3\right)$